

BYMAT

7th BYMAT Conference

Bringing young mathematicians together

University of Sevilla

17-20 November 2025

Contents

Acknowledgements	5
Plenary speakers	9
Paula Gordaliza	9
Alberto Rodríguez	10
Marithania Silvero	11
M ^a Ángeles García-Ferrero	12
Parallel sessions	15
Monday morning	15
Applied Mathematics	15
Statistics	19
Functional Analysis	24
Algebra	28
Monday afternoon	33
Applied Mathematics	33
Operations Research and Optimisation	36
Algebraic Geometry and Number Theory	40
Analysis	43
Tuesday morning	46
Applied Mathematics	46
PDEs and Dynamical Systems	50
Topology	54
Groups	59
Tuesday afternoon	63
Algebra	63
Differential Equations and Discrete Systems	65
Analysis	68
Applied Mathematics	70
Wednesday morning	73
PDEs	73
Convex Geometry	78
Proof Theory	82
Groups	87

Wednesday afternoon	91
Mathematical Foundations	91
Topology	94
Algebraic Geometry and Number Theory	98
Differential Geometry	101
Thursday morning	104
Algebra	104
Algebraic Geometry and Number Theory	106
Operator Theory	109
Topology	111
 Posters	 117
 Committees	 145

Aknowledgements

We want to thank Valladolid 6th BYMAT Conference Organizing Committee (especially Sara Asensio Ferrero and Adrián Fidalgo Díaz), without whom this book of abstracts would have taken way longer to make.

Sponsors



Plenary speakers

Towards Fair and Reliable Machine Learning: Statistical Foundations and Challenges under Uncertainty

Paula Gordaliza, Institute for Advanced Materials and Mathematics (INAMAT²)
Universidad Pública de Navarra (UPNA)

The talk addresses recent advances on the statistical foundations of fair and reliable machine learning, structured around two complementary research directions: achieving fairness in learning algorithms and preserving fairness guarantees under uncertainty and distributional shift.

The first direction focuses on the design of methodologies to mitigate bias in predictive models, both through data preprocessing and fairness-aware learning. At the data level, techniques based on optimal transport and distribution trimming are developed to remove the influence of sensitive variables while retaining relevant predictive information. At the model level, Fair Kernel Regression and Fair Partial Least Squares (PLS) incorporate fairness constraints directly into the optimization framework, relying on covariance operators and reproducing kernel Hilbert space representations to construct predictors that balance accuracy and independence from protected attributes, even in nonlinear or high-dimensional settings.

The second direction explores fairness under uncertainty, where limited, heterogeneous, or evolving data may challenge the stability of fairness guarantees. This line focuses on quantifying and propagating uncertainty in fairness assessment, developing Bayesian inference tools to evaluate whether observed disparities are significant or attributable to random variation. These approaches aim to provide more reliable and interpretable fairness analyses, ensuring that fairness evaluations remain valid under data perturbations and sampling variability.

Symmetry and totally geodesic submanifolds

[Alberto Rodríguez Vázquez](#), Université Libre de Bruxelles

Totally geodesic submanifolds are special submanifolds of a Riemannian manifold characterized by the property that every geodesic within them is also a geodesic of the ambient space where they sit in. They play a central role in Riemannian geometry and their existence is usually related to the existence of symmetries of the ambient space. Because of this, their study is particularly rich in highly symmetric settings, such as those arising from Lie groups and homogeneous spaces. In this talk, I will give a broad overview of results concerning totally geodesic submanifolds in Riemannian manifolds in the presence of symmetry.

Unraveling Knots: Invariants, Homology, and Classification

Marithania Silvero Casanova, Universidad de Sevilla

The problem of classifying knots lies at the heart of knot theory, and invariants have a crucial role when trying to solve this challenge. In this talk, we will present an overview of several families of knot invariants, ranging from classical polynomial and geometric constructions to more recent homological approaches. We will discuss how these invariants encode geometric and topological information about knots and links, with a particular focus on the properties of Khovanov homology and its geometrization.

On the inverse conductivity problem

[María Ángeles García Ferrero](#), ICMAT

The classical Calderón problem is the inverse problem of determining the electrical conductivity of the interior of a medium from voltage and current measurements on its surface. With applications in medical imaging and geophysics, its study has led many developments in the area of inverse problems since the eighties.

In this talk we will introduce some important questions about the Calderón problem and the main ingredients to answer them, focusing on the case of three or higher dimensions and the development concerning the regularity of the conductivity.

Parallel sessions

Robust probability support vector machines with pinball loss

Manuel Gonzalo Carvajal ^a,

Miguel Carrasco ^b, Benjamín Ivorra ^a, Julio López ^c, Angel M. Ramos ^a,

^a, Instituto de Matemática Interdisciplinar, Departamento de Análisis y Matemática Aplicada,
Universidad Complutense de Madrid.

^b, Facultad de Ingeniería y Ciencias Aplicadas, Universidad de los Andes.

^c, Facultad de Ingeniería y Ciencias, Universidad Diego Portales.

Support Vector Machines, SVMs for short, are Machine Learning algorithms mainly used to solve the problem of binary classification. These algorithms work by constructing a classifier function from a set of previously labeled data by solving a convex optimization problem.

Although neural networks are more versatile and effective in many supervised learning applications, they are also prone to overfitting and classifying new observations can have a high computational cost. This makes SVMs more suitable for small databases or deployment in smaller devices.

These advantages have motivated the research in SVM type models as an alternative to neural networks in certain situations. In this context several modifications have been proposed to improve the performance of these algorithms. In our work we focus on two adaptations. First, the probability SVMs presented in [2], that produce a probability as the output instead of just the classification. Second, the utilization of the pinball loss function as suggested by [3], that results in models that are more robust to noise.

In our work we propose a novel SVM model that combines both modifications and obtain the primal and dual form of the corresponding optimization problem. Then we implemented the problem and tested its performance over several databases focusing on the robustness on the solution to perturbation on the input data.

References

- [1] I. Steinwart and A. Christmann (2008) *Support Vector Machines*, Springer, New York.
- [2] Yuan-Hai Shao, Xiao-Jing Lv, Ling-Wei Huang and Lan Bai (2023) *Twin SVM for conditional probability estimation in binary and multiclass classification*, Pattern Recognition.
- [3] Xialoin Huang, Lei Shi and Johan A. K. Suykens (2014) *Support Vector Machine Classifier With Pinball Loss*, IEEE Transactions on Pattern Analysis and Machine Intelligence.
- [4] Miguel Carrasco, Benjamín Ivorra, Julio López and Angel M. Ramos (2025) *Embedded feature selection for robust probability learning machines*, Pattern Recognition.

A Queue Theory approach for modeling computer memory system performance

Mencía Reborido Fuentes, Universidad Autónoma de Madrid (UAM), Instituto de Ciencias Matemáticas (ICMAT)

A memory system is the collection of hardware and processes in a computer that both store data and manage its movement to and from the processor, typically through operations such as read and write commands. In situations of high command traffic, that is, high bandwidth, it is useful to have a mathematical model capable of predicting how the memory system performs under such conditions.

Consequently, in this talk we present a Queue Theory-based model that treats several of the main structures present in a memory system as server-queue components and calculates the total latency of the system (i.e., the time it takes to process a certain bandwidth). In contrast to existing models in the state of the art, we make use of the so-called finite queues which are believed to best adapt to the aforementioned high bandwidth situations. With this implementation, we derive an algorithm that captures the latency delay due to high command traffic in each of the memory components modeled.

References

- [1] Dong-Won, S. (2014). *Explicit Formulae for Characteristics of Finite-Capacity M/D/1 Queues*, ETRI Journal, Vol. 36.
- [2] Hernández-González, S. et al. (2015). *Propiedades de sistemas tipo M/G/1/K con estaciones en serie*, Ingeniería Investigación y Tecnología, vol. XVI no. 3.
- [3] Perros, H.G., Tayfur, A. (1986). *Approximate Analysis of Open Networks of Queues with Blocking: Tandem Configurations*, iee transactions on software engineering, vol. se-12, no.3.

Measuring spatial interactions during stem cell differentiation with TDA techniques

First author, Miguel Navarro Castro

Second author, Elena Camacho-Aguilar and Maria-Jose Jimenez

Stem cells are characterized by their ability to proliferate and differentiate into specialized cell types in response to environmental signals. For example, human pluripotent stem cells differentiate into mesoderm and amniotic cell types in response to BMP (bone morphogenetic protein) signal [1].

To investigate how BMP signalling dynamics affect spatial cell distribution of the different cell types, we apply Topological Data Analysis (TDA) [2] techniques to the sets of 2D points representing different cell types in the final stage of different experiments. More specifically, chromatic alpha complexes [3] have recently emerged as a new construction suitable for the topological analysis of labeled (or chromatic) data. By considering inclusion maps between the chromatic alpha filtrations on subsets of points (for example one-color pointset into the whole set of points), one can derive maps between the homology groups of each filtration value, which can be studied in terms of invariants such as their kernel, cokernel and image persistence diagrams [4]. These persistence diagrams can provide rich information about the spatial relationships between the different pointsets.

This is joint work in progress with Elena Camacho-Aguilar and Maria-Jose Jimenez (named in alphabetical order).

References

- [1] Camacho-Aguilar, E., Yoon, S. T., Ortiz-Salazar, M. A., Du, S., Guerra, M. C., & Warmflash, A. (2024). Combinatorial interpretation of BMP and WNT controls the decision between primitive streak and extraembryonic fates. *Cell systems*, 15(5), 445–461.e4. <https://doi.org/10.1016/j.cels.2024.04.001>
- [2] Dey, T. K., & Wang, Y. (2022). *Computational Topology for Data Analysis*. Cambridge: Cambridge University Press. doi:10.1017/9781009099950
- [3] di Montesano, S. C., Draganov, O., Edelsbrunner, H., & Saghafian, M. (2025). Chromatic alpha complexes. *Foundations of Data Science*. doi:10.3934/fods.2025003
- [4] Cohen-Steiner, D., Edelsbrunner, H., Harer, J., & Morozov, D. (2009). Persistent Homology for Kernels, Images, and Cokernels. In *Proceedings. Proceedings of the 2009 Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)* (pp. 1011–1020). doi:10.1137/1.9781611973068.110

Online Follow-The-Leader (FTL) Model Selection with Conformal Prediction

Nuria Richer Gusano, BCAM – Basque Center for Applied Mathematics

We benchmark hourly probabilistic forecasts selected online by a Follow-The-Leader (FTL) rule over a grid of smoothing and regularization parameters (λ_s, λ_r) . Three selection metrics are compared—RMSE (point error), CRPS (distributional accuracy), and NLL (likelihood fit)—against a baseline model across four datasets. RMSE emphasizes point accuracy; CRPS balances sharpness and calibration; NLL rewards calibrated, often wider, predictive densities. Evaluation uses RMSE, MAPE, CRPS, NLL, and Pinball, with calibration diagnostics from PIT histograms and ECDFs.

Beyond this, we extend the framework with *conformal prediction* to provide finite-sample, distribution-free coverage guarantees on the forecast intervals, ensuring reliability under model misspecification. We also incorporate *dynamic learning* updates, allowing the selection mechanism to adapt not only to cumulative past losses but also to regime shifts and non-stationarities in real time. These enhancements complement FTL by strengthening robustness: conformal layers control coverage, while dynamic adaptation accelerates responsiveness to changing data patterns.

Results show that FTL–CRPS consistently achieves the best probabilistic accuracy (lowest CRPS and Pinball), while FTL–NLL optimizes tail safety (best NLL). Point accuracy remains dataset-dependent. The combined framework highlights a practical trade-off: CRPS for reliable quantiles, NLL for tail robustness, and conformal + dynamic extensions for adaptive, trustworthy deployment in evolving environments.

References

- [1] Gneiting, T., & Raftery, A. E. (2007). *Strictly proper scoring rules, prediction, and estimation*. Journal of the American Statistical Association, 102(477), 359–378.
- [2] Vovk, V., Gammerman, A., & Shafer, G. (2005). *Algorithmic learning in a random world*. Springer.
- [3] Cesa-Bianchi, N., & Lugosi, G. (2006). *Prediction, learning, and games*. Cambridge University Press.

The Galton-Watson Process: Theory and Applications

Carlos Cañada Moreno, Universidad de Extremadura

This talk provides a comprehensive mathematical treatment of the Galton-Watson branching process, a fundamental stochastic model for population dynamics. The process is defined recursively where each individual reproduces independently according to a common offspring distribution $\{p_k\}$, forming a homogeneous Markov chain with transition probabilities given by convolution powers of the reproduction law.

The extinction probability q is characterized as the smallest non-negative root of the equation $f(s) = s$, where f is the probability generating function. The process exhibits distinct asymptotic behaviors across three regimes: subcritical ($m < 1$), critical ($m = 1$), and supercritical ($m > 1$). Critical processes satisfy $\lim_{n \rightarrow \infty} P[z_n/n \leq u \mid z_n > 0] = 1 - \exp(-2u/\sigma^2)$ when $\sigma^2 < \infty$. Supercritical processes exhibit martingale convergence $w_n = z_n/m^n \rightarrow w$ almost surely, with L^1 -convergence if and only if $\sum p_k k \log k < \infty$.

The analysis employs generating functions, martingale theory, and limit theorems to derive precise asymptotic results. Additional topics include hitting time distributions and total progeny analysis, rate of convergence theorems.

References

- [1] Jagers, P. *Branching Processes and Their Applications*. Cambridge University Press, 1985.
- [2] Feller, W. *An Introduction to Probability Theory and Its Applications, Volume II*. Wiley, New York, 1966.
- [3] Harris, T. E. *The Theory of Branching Processes*. Springer, Berlin, 1963.
- [4] Athreya, K. B. and Ney, P. E. *Branching Processes*. Springer-Verlag, Berlin, 1972.
- [5] Kesten, H., Ney, P., and Spitzer, F. *The Galton-Watson process with mean one and finite variance*. Teor. Veroyatnost. i Primenen, 11:579-611, 1966.

Modelling and maintenance planning for repairable systems under multiple degradation processes incorporating imperfect repairs

Emmanuel Moreno Mejía, Universidad de Extremadura

Lucía Bautista Bárcena, Universidad de Extremadura

Inmaculada Torres Castro, Universidad de Extremadura

Reza Ahmadi, Iran University of Science and Technology

This study presents a novel framework for reliability analysis and maintenance planning in structural systems subject to multiple degradation processes. Two failure schemes are addressed. In Failure Scheme I, each degradation mode is described by a Wiener process, and their combined effect is modeled as a linear combination, subject to periodic imperfect maintenance actions according to the $ARD(\infty)$ model. In Failure Scheme II, the degradation processes are modeled as Gamma processes, and a dual failure criterion is introduced: either by total accumulation of degradation or by the exceedance of a threshold associated with a critical degradation process. As in Failure Scheme I, the system is subject to periodic imperfect maintenance actions whose effects are modeled following the $ARD(\infty)$ approach.

For each failure scheme, different maintenance policies based on discrete inspections are proposed. These strategies use both the observed degradation state and estimates of the remaining useful life (RUL) to make optimal decisions between corrective replacement, preventive replacement, or imperfect maintenance, with the objective of minimizing the stationary cost per unit time.

Analytical expressions for the corresponding survival function are also obtained. Subsequently, a numerical study evaluates and compares the efficiency of the proposed maintenance policies in terms of average cost per unit time.

References

- [1] AHMADI, R.(2016) An optimal replacement policy for complex multi-component systems, *International Journal of Production Research*, **54**(17), 5303-5316.
- [2] BAUTISTA, L., CASTRO, I.T. AND LANDESA, L.(2022) Condition-based maintenance for a system subject to multiple degradation processes with stochastic arrival intensity, *European Journal of Operational Research*, **302**(2), 560-574.
- [3] CHUANG C.S.(1996) Joint distribution of Brownian motion and its maximum, with a generalization to correlated BM and applications to barrier options, *Statistics & Probability Letters*, **28**(1), 81-90.
- [4] CABALLÉ, N.C., CASTRO, I.T., PÉREZ, C.J. AND LANZA-GUTIÉRREZ, J.M.(2015) A condition-based maintenance of a dependent degradation-threshold-shock model in a

system with multiple degradation processes, *Reliability Engineering & System Safety*, **134**, 98-109.

Balancing Sparsity and Subgroup Fairness in High-Dimensional Regression: The Cost-Sensitive Constrained Elastic Net

Jaime Díaz-Trechuelo Sánchez-Moliní, Universidad de Sevilla

M^a de los Remedios Sillero Denamiel, Universidad de Sevilla, IMUS

The trade-off among predictive accuracy, interpretability, and subgroup fairness is central in high-dimensional regression. A hallmark of regularized regression is the Lasso [1], which achieves variable selection and sparsity but struggles with highly correlated predictors and offers no mechanism for subgroup fairness. Two extensions to the Lasso address these shortcomings separately: the Elastic Net [2], which modifies the objective function by adding an ℓ_2 term to the Lasso's ℓ_1 penalty, ensuring robustness under multicollinearity; and the CSClasso [3], which modifies the feasible set imposing subgroup-specific bounds on prediction error.

In this talk, we present the Cost-Sensitive Constrained Elastic Net (CSCEN), a novel regularization method developed for my Bachelor thesis that unifies these approaches within a single framework. CSCEN minimizes an Elastic Net-style $(\ell_1 + \ell_2)$ objective *subject to* CSClasso-style fairness constraints, hence providing a new tool that balances accuracy, interpretability, and fairness through error bounds. By construction, the ℓ_2 term guarantees uniqueness of the solution, while the fairness constraints operationalize subgroup equity.

We examine the mathematical properties of the model, prove uniqueness, and establish conditions for consistency. We then present a series of numerical experiments on real and simulated datasets comparing CSCEN to classical methods, highlighting its ability to achieve sparsity while respecting fairness constraints. This work contributes a flexible and robust framework for modern regression tasks where fairness and structural considerations cannot be sacrificed for predictive accuracy alone.

References

- [1] Tibshirani, R. (1996) “Regression shrinkage and selection via the Lasso”. *Journal of the Royal Statistical Society: Series B (Methodological)*, vol. 58(1): 267-288.
- [2] Zou, H. and Hastie, T. (2005) “Regularization and variable selection via the elastic net”. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, vol. 67(2): 301-320.
- [3] Blanquero, R., Carrizosa, E., Ramírez-Cobo, P. and Sillero-Denamiel, M. R. (2020) “A Cost-Sensitive Constrained Lasso”. *Advances in Data Analysis and Classification*, vol. 15(1): 121-158.

Extending TCLUST to higher dimensions

Lucía Trapote Reglero, University of Valladolid

Luis Ángel García Escudero, University of Valladolid

Agustín Mayo Íscar, University of Valladolid

Outliers are known to significantly distort the results of many commonly used clustering methods, often leading to unreliable cluster partitions. To address this issue, different robust clustering approaches have been developed that not only reduce the influence of but also facilitate the detection of meaningful outliers. This presentation focuses on robust clustering methods based on trimming, especially TCLUST, which extends the type of trimming used by MCD in one-population problems to allow for different subpopulations or clusters unknown in advance. While TCLUST performs well on low-dimensional data, it struggles with high-dimensional datasets due to the complexity involved in estimating a large number of parameters. The Robust Linear Grouping (RLG) method offers an alternative by assuming clusters lie near lower-dimensional subspaces, thus combining clustering with dimensionality reduction. However, RLG has limitations when subspaces intersect and assumes simplistic isotropic orthogonal errors. A robust clustering method extending TCLUST, which builds on the High Dimensional Data Clustering (HDDC) method by including trimming and eigenvalue constraints, will be presented. This approach balances TCLUST and RLG, requiring careful adaptation of TCLUST and HDDC steps for proper implementation. An extension allowing for cellwise trimming will also be outlined.

References

- [1] L. A. García-Escudero, A. Gordaliza, R. San Martín, S. Van Aelst y R. Zamar (2009), *Robust linear clustering*, Journal of the Royal Statistical Society. Series B: Statistical Methodology, 71(1), 301–318. DOI:10.1111/J.1467-9868.2008.00682.X.
- [2] C. Bouveyron, S. Girard y C. Schmid (2007), *High-dimensional data clustering*, Computational Statistics & Data Analysis, 52(1), 502–519. DOI:10.1016/j.csda.2007.02.009.
- [3] L. A. García-Escudero, A. Gordaliza, C. Matrán y A. Mayo-Isar (2008), *A general trimming approach to robust cluster analysis*, Annals of Statistics, 36(3), 1324–1345. DOI:10.1214/07-AOS515.

Quasicomplemented subspaces in Banach spaces

Miguel Ángel Ruiz Risueño, UCLM

We review the classical theory about quasicomplemented subspaces of Banach spaces and some of its applications to the geometry of those spaces. We also present several recent results in that direction, especially in the context of Banach spaces with weak star separable dual, some of which have been accomplished in a joint work with Mar Jiménez Sevilla and Sebastián Lajara.

References

- [1] V. P. Fonf, S. Lajara, S. Troyanski and C. Zanco, *Operator ranges and quasicomplemented subspaces of Banach spaces*, Studia Math. **246** (2) (2019), 203–216.
- [2] P. Hájek, V. Montesinos, J. Vanderwerff, and V. Zizler, *Biorthogonal systems in Banach spaces*, CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, vol. 26, Springer, New York, 2008.
- [3] M. Jiménez-Sevilla and S. Lajara, *Quasicomplemented subspaces of Banach spaces and separable quotients*, Results Math. **78** (6) (2023), paper No. 244, 24 pp.
- [4] W. B. Johnson, *On quasicomplements*, Pacific J. Math. **48** (1973), 113–118.
- [5] J. Lindenstrauss, *On subspaces of Banach spaces without quasicomplements*, Israel J. Math. **6** (1968), 36–38.

Interpolation of Banach spaces and uniform homeomorphism of spheres

Giulia Fantato, Institute of Mathematics and Statistics, University of São Paulo

A Banach space is simultaneously a vector space and a metric space, which is why they are usually clasified by linear homeomorphisms, as these preserve both structures. A fundamental question, however, is to what extent the metric structure already determines the linear one. In other words, we are interested in classifying Banach spaces via non-linear homeomorphisms.

One approach is to study uniform homeomorphisms between the unit spheres of Banach spaces. A central open problem in this direction asks: Is the unit sphere of every separable, uniformly convex, infinite-dimensional Banach space uniformly homeomorphic to the unit sphere of ℓ_2 ? This problem remains unsolved in general, though it has a positive answer for Banach spaces with a lattice structure.

However, Daher [1] showed that the method of complex interpolation can be a powerful tool in this context: under suitable uniform convexity assumptions on the boundary spaces, the spheres of all interpolation spaces are uniformly homeomorphic.

References

- [1] M. Daher (1995) *Homéomorphismes uniformes entre les sphères unité des espaces d'interpolation*, Canadian Mathematical Bulletin, 38, 286–294.

Projective characterizations of Lindenstrauss and Gurarii spaces

Esteban Martínez Vañó, Universidad de Granada

The Hahn-Banach theorem is a cornerstone of Functional Analysis and the search for a vector-valued version of this result leads to the concept of injective Banach spaces. However, this class is quite restrictive. To extend the theory to a broader context, one can consider two other classes of Banach spaces whose extension properties are related to finite-rank operators: L_1 **preduals** and **Gurarii spaces**. These classes can be characterized by specific projective properties involving the classical notion of **ideal** and its **(almost) isometric** counterpart, respectively.

Well-established generalizations of L_1 preduals and Gurarii spaces exist, namely κ **injective spaces** and spaces of **(almost) universal disposition**. A natural question is whether these broader classes can also be described by analogous projective properties. The answer is affirmative and leads to the development of **transfinite versions** of the concepts of (almost isometric) ideals.

In this talk, we will describe these new transfinite notions and explain how they characterize the corresponding classes of generalized L_1 preduals and Gurarii spaces. Furthermore, we will present constructions of examples that fall outside these classical types, which were the original motivation for this new framework.

References

- [1] E. Martínez Vañó and A. Rueda Zoca, Transfinite (almost isometric) ideals in Banach spaces, arXiv:2505.04178

A brief introduction to APEPs

Juan Guerrero-Viu, Universidad de Zaragoza

Extreme points play a fundamental role in the study of convex sets. In particular, they provide useful information about the geometry and structure of the unit ball of Banach spaces. Among them, denting points are especially significant, as their existence characterizes the Radon–Nikodým Property (RNP). Halfway between these two notions are preserved extreme points which refine extremality while also reveal a rather better interplay with the RNP.

In this talk, we introduce a further weakening of this last concept: the notion of *almost preserved extreme point* (APEP). We will define this new class of points and present a complete description of the APEPs of the unit ball in several classical Banach spaces, illustrating the relevance and potential of this concept.

This is part of a joint work with Ramón J. Aliaga, Luis C. García-Lirola, Matías Raja and Abraham Rueda Zoca.

All Solid Rings

Jaime Benabent Guerrero, Universidad de Sevilla

We give an explicit characterization of all the solid rings, refining in this way a previous work by Bousfield and Kan in the 70s. As a consequence of our methods, we give criteria for finding the core of a ring and explicitly compute the core of some rings.

References

- [1] A. K. Bousfield and D. M. Kan (1972) *Homotopy Limits, Completions and Localizations*, Lecture Notes in Mathematics 304, Springer Berlin, Heidelberg
- [2] J. Benabent Guerrero (2025) *All Solid Rings* <https://arxiv.org/pdf/2502.02484>

The hidden geometry in the system $Ax = b$

Diego Alba Alonso, Departamento de Matemáticas, ETSII, Universidad de Castilla-La Mancha,
13071, Ciudad Real, Spain

Any system of m linear equations (with coefficients in a field k) and n unknowns can be thought as a matrix system $Ax = b$ where $A \in \text{Mat}_{m \times n}(k)$ (is a $m \times n$ matrix with entries in k), $x \in \text{Mat}_{n \times 1}(k)$ and $b \in \text{Mat}_{m \times 1}(k)$. It is a well known fact that the solutions of this system form an affine manifold. However, there is another connection between the solutions of this system and affine geometry which arises from the use of generalized inverses. In this talk we will explore this connection.

Understanding the Pedagogical Content Knowledge behind the elaboration of an advanced algebra textbook

Ignacio Duro Caballero, Universidad Complutense de Madrid

Eric Flores Medrano, Universidad Complutense de Madrid

Nuria Joglar Prieto, Universidad Complutense de Madrid

For decades, several studies have argued that mathematics teachers' knowledge is specialized—that is, qualitatively different from the knowledge held by other professionals who use mathematics and by teachers of other disciplines (Climent & Montes, 2022). Within this specialized knowledge, a consolidated and central construct in mathematics education research is Pedagogical Content Knowledge (PCK) (Shulman, 1986), which refers to elements of teachers' knowledge about the mathematical content to be taught that go beyond the content itself. This domain includes aspects that may facilitate or hinder learning, as well as specific tools, representations, and techniques used to support teaching (Star, 2023).

The aim of this talk—based on my Master's thesis—is to present a general framework of elements of specialized knowledge that allows us to better understand the PCK of an expert university professor. To this end, we propose a qualitative study, specifically an intrinsic case study (Kvale, 1996), which offers an analysis of a mathematics textbook on Algebraic Equations authored by our subject of study and addressed primarily to third-year undergraduate mathematics students. For this purpose, the Mathematics Teacher's Specialised Knowledge model (Carrillo et al., 2018) will be employed, adopting an interpretative approach aimed at identifying, organizing, and interrelating evidence of specialized knowledge mobilized by the author in the process of writing the textbook found in analyzed text excerpts.

References

Carrillo-Yáñez, J., Climent, N., Montes, M., Contreras-González, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, Á., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, 20(3), 236–253. <https://doi.org/10.1080/14794802.2018.1479981>

Climent, N., & Montes, M. Á. (2022). El modelo MTSK: antecedentes y estructura [Capítulo]. En A. Aguilar González, L. J. Rodríguez-Muñiz, & L. Muñiz-Rodríguez (Eds.), *Investigación sobre conocimiento especializado del profesor de matemáticas (MTSK): 10 años de camino* (pp. 27–34). Dykinson. <https://doi.org/10.14679/1451>

Kvale, S. (1996). Interviews: An introduction to qualitative research interview-

ing. SAGE.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14. <http://www.jstor.org/stable/1175860>

Star, J. R. (2023). Revisiting the origin of, and reflections on the future of, pedagogical content knowledge. *Asian Journal for Mathematics Education*, 2(2). <https://doi.org/10.1177/27527263231175885>

A Control Theory for Monomial Dynamical Systems over Finite Fields

Dennis Edgardo Quintano Villanueva, University of Puerto Rico, Mayagüez Campus

Omar Colón Reyes, University of Puerto Rico, Mayagüez Campus

An open problem in the theory of discrete dynamical systems is determining when a system can be "stabilizable." These systems have applications in cellular automata, biology, and engineering. Criteria for determining when a monomial dynamical system over a finite field is a fixed point system (FPS) have been previously studied. These results can be used to determine when a monomial dynamical control system is stabilizable, meaning its behavior can be influenced to achieve a desired goal. In this work, we present a family of control systems that can be stabilized by applying linear algebra over finite fields and provide the necessary conditions under which such systems, in general, can be stabilized. These findings offer a deeper understanding of stabilizability in discrete systems with potential applications across multiple scientific and engineering disciplines.

References

- [1] D. Bollman, O. Colón-Reyes, V. A. Ocasio, and E. Orozco (2010) *A Control Theory for Boolean Monomial Dynamical Systems*, Discrete Event Dynamic Systems, 20:19–35. Springer.
- [2] O. Colón-Reyes, A. S. Jarrah, R. Laubenbacher, and B. Sturmfels (2006) *Monomial Dynamical Systems over Finite Fields*, Complex Systems, 16:333–342.
- [3] O. Colón-Reyes, D. Bollman, and E. Orozco (2007) *Fixed Points in Discrete Models for Regulatory Genetic Networks*, EURASIP Journal on Bioinformatics and Systems Biology.
- [4] B. Pareigis, O. Colón-Reyes, and R. Laubenbacher (2004) *Boolean Monomial Dynamical Systems*, Annals of Combinatorics, 8:425–439.
- [5] D. Panario and G. L. Mullen (2013) *Handbook of Finite Fields*. Discrete Mathematics and Its Applications. CRC Press.

Difference equation for quasi-orthogonal polynomials related to a general operator

Cristina Rodríguez Perales, Universidad de Almería, Spain

Galina Filipuk, University of Warsaw, Poland

Juan F. Mañas Mañas, Universidad de Almería, Spain

Juan J. Moreno Balcázar, Universidad de Almería, Spain and Instituto Carlos I de Física Teórica y Computacional, Spain

The aim of this work is to obtain the second-order difference equation satisfied by the first-order quasi-orthogonal polynomials $s_n(x)$, given by

$$s_n(x) = p_n(x) + c_n p_{n-1}(x),$$

where $p_n(x)$ are the orthonormal polynomials related to the Hahn operator, and c_n is a sequence of real numbers. In order to achieve that, we begin by deriving the ladder operators for the polynomials $p_n(x)$, which allow us to establish the second-order difference equation for these polynomials and generalize some known results in the literature. Finally, these results are illustrated for two specific families of polynomials: the quasi-Krawtchouk and the quasi-little q -Laguerre polynomials.

References

- [1] Y. Chen, M.E.H. Ismail (1997). Ladder operators and differential equations for orthogonal polynomials. *J. Phys. A: Math. Gen.*, 30 , 7918–7829.
- [2] G. Filipuk, J. F. Mañas-Mañas, J. J. Moreno-Balcázar, C. Rodríguez-Perales. Second-order difference equation for quasi-orthogonal polynomials related to Hahn difference operator. *Submitted*.
- [3] M.E.H. Ismail, X.S. Wang (2019). On quasi-orthogonal polynomials: Their differential equations, discriminants and electrostatics. *J. Math. Anal. Appl.*, 474 , 1178–1197.

A general theory of nonlocal elasticity based on nonlocal gradients

Guillermo García-Sáez, Universidad de Castilla-La Mancha

Bessel potential spaces have gained renewed interest due to their robust structural properties and applications in fractional partial differential equations (PDEs). These spaces, derived through complex interpolation between Lebesgue and Sobolev spaces, are closely related to the Riesz fractional gradient introduced by Shieh and Spector in [6, 7]. In [3], the equations of nonlocal nonlinear elasticity based on those gradients are studied and related with the well-known Eringen's model. Recently, a broader class of nonlocal gradients have been introduced based on general kernels in [4, 5] that include the particular case of the Riesz fractional gradient. In this talk we present the results obtained in [1, 2] in which we derive the equations of nonlinear elasticity based on the nonlocal gradients for general kernels. Furthermore, we perform a formal linearization of the equations to obtain the linear equations based on those nonlocal gradients. We prove existence and uniqueness of solutions providing a general nonlocal Poincaré and Korn's inequality using a translation operator from the nonlocal Bessel potential spaces to the classical Sobolev spaces. We also study the connection with the Eringen's model in the most general setting and the localization of the equations for varying horizons.

References

- [1] J. C. Bellido, J. Cueto, and G. García-Sáez. Compact embeddings of Bessel potential spaces. Preprint, arXiv:2506.01677, 2025.
- [2] J.C. Bellido and G. García-Sáez *A general theory of nonlocal elasticity based on nonlocal gradients and connections with Eringen's model*, On preparation.
- [3] J. C. Bellido, J. Cueto, and C. Mora-Corral. Eringen's model via linearization of nonlocal hyperelasticity. *Mathematics and Mechanics of Solids*, 29(4):686–703, 2023.
- [4] J. C. Bellido, C. Mora-Corral, and H. Schönberger. Nonlocal gradients: Fundamental theorem of calculus, Poincaré inequalities and embeddings. Preprint, arXiv:2402.16487, 2024.
- [5] J. Cueto, C. Kreisbeck, and H. Schönberger. Γ -convergence involving nonlocal gradients with varying horizon: Recovery of local and fractional models. *Nonlinear Analysis: Real World Applications*, 85:104371, 2025.
- [6] T. Shieh and D.E. Spector. On a new class of fractional partial differential equations I. *Advances in Calculus of Variations*, 8(4):321–336, 2015.
- [7] T. Shieh and D.E. Spector. On a new class of fractional partial differential equations II. *Advances in Calculus of Variations*, 11(3):289–307, 2018.

The Power Series Method and Its Implementation Using Automatic Differentiation

[Paula Vázquez García](#), Universidade de Santiago de Compostela

Rodrigo López Pouso, CITMAga & Universidade de Santiago de Compostela

Francisco Javier Fernández Fernández, CITMAga & Universidade de Santiago de Compostela

The aim of this talk is to delve into the theoretical foundations of the well-known power series method and its novel application to the lower and upper solutions method. To do so, we develop two programs in Python. The first program enables the construction of the Taylor polynomial of the desired degree for the solution of an initial value problem associated with an ordinary differential equation, under the assumption that the given function is analytic - a case in which the effectiveness of the power series method is ensured. The second program addresses a boundary value problem of Dirichlet type associated with an ODE with a continuous given function. This program does not aim to approximate the solution, but rather to prove its existence by constructing both a lower and an upper solution for the problem.

References

- [1] Abad, A., Barrio, R., Blesa, F., Rodríguez, M. (2012). Algorithm 924: TIDES, a Taylor series integrator for differential equations. *ACM Trans. Math. Softw* **39**(1), Article 5.
- [2] De Coster, C., Habets, P. (2006). *Two-Point Boundary Value Problems: Lower and Upper Solutions*. Vol. 205 of Mathematics in Science and Engineering. Elsevier.
- [3] Margossian, C.C. (2019). A review of automatic differentiation and its efficient implementation. *WIREs Data Mining Knowl Discov*.
- [4] Piccinini, L.C., Stampacchia, G., Vidossich, G. (1984). *Ordinary Differential Equations in $\mathbb{R}^n$: Problems and Methods*. Springer.

A centrality-based GRASP to the Perfect Awareness Problem.

Cristian Pérez-Corral, Universitat Politècnica de València

The spread of information on social networks has been intensively studied across domains such as politics, marketing, and health. Within this context, influence maximization problems—aiming to identify a set of users (seeds) that maximize information reach—have received significant attention. We focus on the perfect awareness problem, a related challenge that seeks the smallest set of seeds capable of making the entire network aware of specific information. Prior approaches include metaheuristics [1], exact integer linear optimization [2], and heuristics based on network topology [3]. In this work, we propose a Greedy Randomized Adaptive Search Procedure (GRASP) enhanced with a centrality-based heuristic. Our method achieves results comparable to state-of-the-art approaches while offering an efficient and scalable solution.

References

- [1] Pereira, Felipe de C. and de Rezende, Pedro J. and de Souza, Cid C. (2021) *Effective Heuristics for the Perfect Awareness Problem*, Procedia Computer Science.
- [2] Pereira, Felipe de C. and de Rezende, Pedro J. and de Souza, Cid C. (2022) *A computational study of the Perfect Awareness Problem*, Procedia Computer Science.
- [3] Gautam, Rahul Kumar and Kare, Anjeneya Swami and Bhavani, S. Durga. (2023) *Centrality Measures Based Heuristics for Perfect Awareness Problem in Social Networks*, Proceedings of Multi-disciplinary Trends in Artificial Intelligence (LNCS).

Mathematical optimization models for constructing multiperiod self-amplifying subhypernetworks

Juan Francisco Ocaña Rivas, Universidad de Granada

Víctor Blanco, Universidad de Granada

Ricardo Gázquez, Universidad de Granada

Networks are a powerful tool to model systems composed of entities and their interactions. Classical graph-based models, however, are inherently pairwise and fail to capture multi-way relationships present in many real-world systems. Hypergraphs, by allowing hyperarcs connecting multiple nodes, provide a richer framework, crucial in contexts such as chemical reaction networks, multi-agent systems, and production processes. Despite their expressive power, hypergraphs remain comparatively underexplored from the combinatorial optimization perspective.

In this work, we focus on the construction and analysis of *self-amplifying subhypergraphs* within multi-directed hypergraphs, recently introduced in [2], using mathematical optimization tools. These substructures, inspired by autocatalytic sets in chemistry [1, 3, 4] and Von Neumann growth models in economics, formalize the idea of subsystems capable of sustaining and amplifying themselves through internal interactions. To measure this phenomenon, we employ the notion of the *maximal amplification factor* (MAF), which quantifies a hypergraph's capacity for self-sustained growth.

Building upon these ideas, the main contribution of this work is the development of new mathematical optimization models for the *multiperiod* setting. We design discrete optimization formulations, under different modeling assumptions, to simulate the temporal evolution of hypernetworks and identify self-amplifying substructures across multiple time periods. These formulations range from nonlinear fractional programs related to generalized eigenvalue problems to mixed-integer linear and nonlinear programming models, allowing us to capture both structural constraints and dynamical aspects. Additionally, we will discuss on the stochastic nature of this type of networks and how to consider it into the mathematical models that we propose.

Beyond their theoretical interest, these models have direct applications. They provide tools for the visualization of higher-order network evolution and, in the case of chemical reaction networks (CRNs), can incorporate kinetic information to study amplification processes relevant to questions on the *Origin of Life* [5].

Our approach highlights the interplay between discrete optimization, hypergraph theory, and dynamical systems. The multiperiod perspective opens new directions for the systematic study of evolving hypernetworks, combining rigorous mathematical formulations with application-driven insights.

References

- [1] Andersen, Jakob L. and Flamm, Christoph and Merkle, Daniel and Stadler, Peter F., *Chemical Transformation Motifs-Modelling Pathways as Integer Hyperflows*, Natural Computing, 2019, 18(1), 91–109.
- [2] Blanco, Víctor and González, Gabriel and Gagrani, Praful, *Identifying Self-Amplifying Hypergraph Structures through Mathematical Optimization*, arXiv preprint arXiv: 2412.15776, 2025.
- [3] Blokhuis, Alex and Lacoste, David and Nghe, Philippe, *Universal motifs and the diversity of autocatalytic systems*, Proceedings of the National Academy of Sciences, 2020, 117(41), 25230–25236.
- [4] Gagrani, Praful and Skoulakis, Spyros, *The geometry of autocatalytic sets*, Journal of Mathematical Biology, 2023, 87, 1–37.
- [5] Hordijk, Wim and Steel, Mike, *Autocatalytic networks at the basis of life's origin and organization*, Life, 2018, 8(4), 62.

The total distance dominating set problem

L. Cruz, Department of Economics, Quantitative Methods, and Economic History, University Pablo de Olavide, Spain

A.D. López-Sánchez, Department of Economics, Quantitative Methods, and Economic History, University Pablo de Olavide, Spain

E. Barrena, Department of Economics, Quantitative Methods, and Economic History, University Pablo de Olavide, Spain

Domination problems in graphs have been extensively studied over the past decades due to their broad applicability in real-world scenarios. Specifically, the Minimum Dominating Set Problem (MDSP)[1] aims to identify the smallest subset of nodes such that every node not included in the subset is adjacent to at least one node within it. Practical applications of this problem include service location, communication network design, and social network analysis. However, the classical domination problem is defined on unweighted graphs and does not consider the distances between dominating nodes and the nodes they dominate. As a result, some solutions may include dominated nodes that are located far from their respective dominating nodes, leading to inefficient connections in terms of cost, time, or quality.

The Total Distance Dominating Set Problem (TDDSP) focuses on achieving a trade-off between minimizing the size of the dominating set and minimizing the sum of the distances between the dominating nodes and the nodes they dominate. By considering both criteria, this problem seeks to obtain higher-quality solutions that are more efficient in practical applications.

The TDDSP is a bi-objective problem and, since the MDSP is NP-hard, the TDDSP is also NP-hard. To solve it, a metaheuristic based on Variable Neighborhood Search (VNS) [2] has been implemented. This procedure is design to overcome the issue of getting stuck in a local minima by automatically changing neighbourhoods during the search.

References

- [1] E.J. Cockayne and S.T. Hedetniemi. Towards a theory of domination in graphs. *Networks*, 7:247–261, 1977.
- [2] P. Hansen, N. Mladenovic, and J. Moreno-Pérez. Variable neighbourhood search: Methods and applications. *JOR*, 175:367–407, 02 2010.

Counting Points on Genus 3 Superelliptic Curves over Finite Fields

[Francesc Pedret](#), Universitat Politècnica de Catalunya

Francesc Fité, Universitat de Barcelona

Xavier Guitart, Universitat de Barcelona

We focus on the classical problem of counting points on algebraic curves over finite fields. The well-known Weil conjectures, proven by Weil for curves and by Deligne for general abelian varieties, show that the a priori infinite problem of determining the number of points of a curve C over all finite fields of a fixed characteristic p can be reduced to a finite problem. More precisely, the Zeta function of the curve at p , whose logarithmic derivative is a generating function for $\#C(\mathbb{F}_{p^n})$, turns out to be a rational function whose only unknown term is the numerator, called the L -polynomial of C at p and denoted by $L_p(C, T)$. This motivates the development of efficient algorithms to compute $L_p(C, T)$.

In this talk, we describe a new practical algorithm to compute $L_p(C, T)$ when C admits an affine model $y^3 = x^4 + ax^2 + bx + c$ or $y^2 = x^7 + ax^5 + bx^3 + cx$. In particular, we are able to compute $L_p(C, T)$ for all primes p up to some bound N with complexity $\mathcal{O}(N \log(N)^{3+o(1)})$.

Exact Autonomous Identities for the Sum of Primes and the Prime-Counting Function

Adrián Macías Quintero, Universidad de Sevilla, Spain

We present new exact identities linking the prime-counting function $\pi(n)$ with the sum of primes $\sum_{p \leq n} p$, derived from novel reformulations of Wilson's theorem. Our central result, valid for $n \geq 4$, provides the first completely autonomous closed formula that relates these fundamental functions without requiring primality testing or precomputation:

$$\sum_{p \leq n} p = \pi(n) - 2 + \sum_{i=2}^n \left[(i-1)! - i \left\lfloor \frac{(i-1)!}{i} \right\rfloor \right].$$

We further derive a second independent expression for $\sum_{p \leq n} p$ (valid for $n \geq 6$) and show how both identities lead to an independent derivation of the Hardy-Wright factorial representation of $\pi(n)$. This significantly advances the classical theory by providing a unified framework that connects additive and multiplicative aspects of prime numbers through elementary combinatorial methods.

Beyond prime sums, we extend this approach to obtain novel exact autonomous formulas for prime reciprocal sums $\sum_{p \leq n} p^{-k}$ ($k > 0$), providing finite counterparts to classical asymptotic results of Mertens. These discoveries demonstrate that prime counting, prime summing, and reciprocal relations all emerge from a common factorial structure.

Although computationally intractable for large n due to factorial growth, these identities offer substantial theoretical and pedagogical value by revealing new connections between prime counting and summing functions within an elementary arithmetic framework.

References

- [1] A. Macías-Quintero (2025) *Identidades autónomas y exactas en teoría de números*, Zenodo, preprint, <https://doi.org/10.5281/zenodo.17509229>
- [2] F. Mertens (1874) "Ein Beitrag zur analytischen Zahlentheorie," *Journal für die reine und angewandte Mathematik*, 78, 46–62
- [3] G. H. Hardy and E. M. Wright (2008) *An Introduction to the Theory of Numbers*, 6th edition, Oxford University Press.
- [4] T. Nagell (1951) *Introduction to Number Theory*, Wiley, New York, pp. 99-101.

Polynomial bases and homomorphisms on representations of the symmetric group

Aarón Ocampo Amaya, Universidad de Sevilla

This talk/poster is meant to show some original results based on well known constructions in the field of representation theory. We intend for this presentation to be understandable from an entry level point of view. We'll take the time to introduce the theory and motivate the results; avoiding the more technical details. A summary of the presentation is now given.

Polynomial bases for representations of the symmetric group $\mathbb{S}_n$ have been studied extensively since the introduction of higher Specht polynomials [1]. These polynomials serve as a nice way of both computationally working with representations, as well as, showing the fundamental combinatorial aspects underlying the theory of $\mathbb{S}_n$ representations.

The regular Specht polynomials can be constructed as a polynomial base for the irreducible component V_λ in the space of polytabloids M_λ for each partition $\lambda \vdash n$ as in [2]. However, as these polytabloid spaces M_λ contain other irreducible components V_μ for $\mu \supseteq \lambda$; they must also contain polynomial bases for these other irreducible components. While similar, these are different from the higher Specht polynomials.

This work explicitly computes these other new polynomials, which turn out to be products of the regular Specht polynomials and certain Schur polynomials. Multiplying by these Schur polynomials, now gives homomorphisms of $\mathbb{S}_n$ -representations $V_\mu \longrightarrow M_\lambda$.

We then use these results to compute homomorphisms between $\mathbb{S}_n$ - and $\mathbb{S}_{n+1}$ - representations, on some simple cases. These homomorphisms appear in the theory of FI-modules introduced by Church-Ellenberg-Farb in [3]. While the main theory of FI-modules focuses on studying stability on some representation sequences, we hope to expand this idea onto a more general setting; that gives another interpretation of the "hook stability" portrayed in [4].

References

- [1] E. Briand, A. Rattan, and M. Rosas. On the growth of the kronecker coefficients, ArXiv e-prints, July 2016, 1607.02887.
- [2] T. Church, Jordan S. Ellenberg, and B. Farb. FI-modules and stability for representations of symmetric groups. *Duke Mathematical Journal*, 164(9):1833 – 1910, 2015.
- [3] B. Sagan. *The symmetric group: representations, combinatorial algorithms, and symmetric functions*, volume 203. Springer Science & Business Media, 2001.
- [4] T. Terasoma and H. Yamada. Higher specht polynomials for the symmetric group. *Proc. Japan Acad. Ser. A Math. Sci.*, 69(10):41–44, 1993.

Phase retrieval by example

Jesús Illescas-Fiorito, UCM-ICMAT

Phase retrieval concerns the reconstruction of scalar-valued functions from measurements in which only modulus information is available, a problem that arises naturally in many applied mathematics contexts. In this talk, we explore how the difficulty of this problem, when formulated in the abstract setting of Banach lattices, changes dramatically when moving from the real-valued to the complex-valued case. Our goal is to present these ideas with numerous illustrative examples, as well as some open (and hopefully interesting) questions.

Joint project with Teresa Luque (UCM) and Pedro Tradecete (ICMAT).

References

- [1] D. Freeman, T. Oikhberg, B. Pineau, M. A. Taylor (2023) *Stable phase retrieval in function spaces*, *Mathematische Annalen*, 390(1), 1–43.
- [2] J. Illescas-Fiorito (2025) *Phase retrieval in Banach lattices*, Master's Thesis, UCM. Available online: <https://sites.google.com/ucm.es/jillesca-math>

Unique Preduals in Banach Spaces

[Mario Guillén](#), Universitat Politècnica de València

Pedro Tradacete, Instituto de Ciencias Matemáticas (ICMAT)

The problem of a Banach space admitting a unique predual is a fundamental problem in functional analysis. This work explores this problem, with an emphasis on spaces of bounded and holomorphic functions. We review the state of the art and present techniques such as property (X) and L-embeddedness, which prove useful in establishing uniqueness. The core of the talk focuses on the space $H^\infty(\mathbb{D})$ of bounded and holomorphic functions on the complex disk. After presenting the general existence of preduals, we revisit Ando's theorem of uniqueness of predual of this space, filling gaps left in the literature. We then extend Ando's uniqueness theorem to the case where the domain is a disjoint union of simply connected open sets. Finally, we discuss the obstacles that arise in an attempt to extend these results to the higher-dimensional setting.

References

- [1] T. Ando. "On the predual of H^∞ ." In: *Commentationes Mathematicae: Tomus Specialis in Honorem Ladislai Orlicz 1* (1978), pp. 33-40.
- [2] G. Godefroy. "Existence and uniqueness of isometric preduals: A survey". In: *Banach space theory (Proc., Iowa City, 1987)*. Ed. by Bor-Luh Lin. Vol. 85. Contemporary Mathematics. Providence, RI: American Mathematical Society, 1989, pp. 131-193.
- [3] P. Harmand, D. Werner, and W. Werner. *M-Ideals in Banach Spaces and Banach Algebras*. Lecture Notes in Mathematics. Springer Berlin Heidelberg, 2006.
- [4] R. Aron, J. Bonet, and M. Maestre. "Norm Attaining Elements of the Ball Algebra $H^\infty(B_N)$ ". In: *Results in Mathematics* 79 (2024).

Characterizing the boundedness of the segment multiplier in rearrangement-invariant spaces

Miguel F. Barea-Fernández, Universidad Complutense de Madrid

Jan Lang, Ohio State University

Javier Soria, Universidad Complutense de Madrid

The segment multiplier and the Hilbert transform are two operators that have been closely linked in the context of L^p spaces, where it turns out that they share the same norm for $1 \leq p \leq \infty$, being both unbounded at $p = 1, \infty$ [2]. This fact leads to the following natural question: do both operators always have the same norm?

To study this question, we focus on rearrangement-invariant spaces, a general class that includes Lebesgue spaces, Orlicz spaces and weighted Lorentz spaces. In these spaces, the boundedness of the Hilbert transform is characterized by a simple inequality involving a pair of numbers associated to each space, called Boyd indices [1].

Actually, the answer to the question is negative: there exist spaces where the segment multiplier is bounded but the Hilbert transform is not. Then, our next concern is if we can give another simple condition, similar to that of the Hilbert transform, that characterizes the boundedness of the segment multiplier.

In this talk, we introduce a method to obtain two modified Boyd indices that give this desired characterization, and show with some examples that it is strictly weaker than the one for the Hilbert transform.

References

- [1] David W. Boyd (1967), “The Hilbert transform on rearrangement-invariant spaces”, *Canadian J. Math.* vol. 19, pp. 599-616.
- [2] Laura De Carli and Enrico Laeng (2000), “Sharp L^p estimates for the segment multiplier”, *Collect. Math.* vol. 51 (3), pp. 309-326.

When Mathematics Meets Emotions: A Fuzzy Logic approach to Human Affective Processing

Amaia Gastearena Irigoyen, Universidad Pública de Navarra

The study of human emotions often involves complex, uncertain, and multidimensional data. Traditional statistical approaches face limitations when modeling these types of phenomena, where subjective perception and variability play a central role. In this work, we present an application of fuzzy logic as a mathematical tool to model affective responses in experimental settings.

The proposed methodology is developed within a Virtual Reality-based Behavioral Biomarker (VRBB) framework, originally designed to analyze how different sensory stimuli—such as audiovisual and olfactory inputs—influence emotional states in virtual reality. Using fuzzy sets and rule-based systems, we address the uncertainty inherent in human responses and generate interpretable models that capture relationships between inputs (stimuli) and outputs (affective evaluations).

Our contribution highlights how fuzzy modeling—an approach rooted in fuzzy logic—can both test predefined hypotheses regarding human affective processing and discover new fuzzy rules emerging from data. This example illustrates the potential of mathematical methods, showing how fuzzy logic—still rarely applied in psychology—can bridge experimental data with expert-driven insights.

References

- [1] Sergio Cervera-Torres, Maria Eleonora Minissi, Alberto Greco, Alejandro Callara, Saideh Ferdowsi, Luca Citi, Luna Maddalon, Irene Alice Chicchi Giglioli, and Mariano Alcañiz (2023). *Modulating virtual affective elicitation by human body odors: advancing research on social signal processing in virtual reality*. In: *International Conference on Human-Computer Interaction*, pp. 317–327. Springer.
- [2] Lotfi A. Zadeh (1965). *Fuzzy sets*. Information and Control, 8(3), 338–353. doi:10.1016/S0019-9958(65)90241-X
- [3] T. Takagi and M. Sugeno (1985). *Fuzzy identification of systems and its applications to modeling and control*. IEEE Transactions on Systems, Man, and Cybernetics, 15(1), 116–132. doi:10.1109/TSMC.1985.6313399
- [4] Segismundo S. Izquierdo and Luis R. Izquierdo (2018). *Mamdani Fuzzy Systems for Modelling and Simulation: A Critical Assessment*. Journal of Artificial Societies and Social Simulation, 21(3), 2. doi:10.18564/jasss.3660

Mathematical modeling and numerical simulation of batteries

Eva Garijo Alcalde, Instituto de Matemática Interdisciplinar, UCM (evgarijo@ucm.es)

Juan Antonio Infante del Río, Instituto de Matemática Interdisciplinar, UCM

Benjamin Ivorra, Instituto de Matemática Interdisciplinar, UCM

Ángel Manuel Ramos del Olmo, Instituto de Matemática Interdisciplinar, UCM

Over the past few years, lithium-ion batteries have become a key technology for energy storage, proving highly useful in diverse applications ranging from portable electronic devices to electric vehicles and large-scale storage systems. Accurate modeling of these batteries is crucial to improving their performance, extending their lifetime, and ensuring their safety. Various approaches exist for mathematically modeling lithium-ion batteries, including electrochemical, equivalent circuit, and data-driven models. Each method has particular characteristics and is applied in specific contexts.

This short talk presents the studies and results derived from work employing electrochemical models, which offer a detailed description of the physical and chemical processes occurring inside a battery [1]. These models consider phenomena such as ion transport, intercalation/deintercalation reactions, and the formation of SEI (solid electrolyte interphase) layers on the electrodes. One of the main advantages of electrochemical models is their ability to offer detailed insights into the internal behavior of the battery, making them ideal for the analysis and optimization of cell and material design.

Some of their main features include high accuracy in representing internal phenomena, balanced by the need for detailed knowledge of material parameters and properties. The simulations presented here have been obtained using the solvers *DandeLiion* [2], a fast solver for the Newman model, and *COMSOL Multiphysics* [3] for comprehensive multiphysics modeling.

References

- [1] G-W. Richardson, J.M. Foster, R. Ranom, C.P. Please & A. M. Ramos, *Charge transport modelling of Lithium-ion batteries*, European Journal of Applied Mathematics , Volume 33 , Issue 6 , December 2022 , pp. 983 - 1031.
- [2] I. Korotkin, S. Sahu, S. E. J. O’Kane, G. Richardson, and J. M. Foster, "DandeLiion v1: An Extremely Fast Solver for the Newman Model of Lithium-Ion Battery (Dis)charge," *Journal of The Electrochemical Society*, vol. 168, no. 6, p. 060544, Jun. 2021.
- [3] COMSOL, *COMSOL Multiphysics*, Version 6.3, COMSOL Inc., 2024.

Dynamic and numerical methods of average theory with application in oscillator problems

Jorge Rodríguez Pérez, Universidad de Valladolid

This work aims to demonstrate various bounds for the difference between the solution of a system of differential equation where two time-scales are present and its averaged system, using a modern and rigorous approach. The concept of averaging is defined, and properties of KBM functions are analyzed, establishing bounds for the difference between solutions on a time scale of $1/\varepsilon$. Additionally, UKBM functions are introduced to generalize results to unbounded intervals. The theory of averaging for periodic functions is presented in its classical version, studying a change of variables that transforms the original equation into the averaged one, and generalizing to higher-order approximations. Finally, practical applications are presented, such as the analysis of the Kapitza pendulum and the theoretical basis and implementation of the of stroboscopic averaging numerical methods, which allow for the efficient integration of oscillator problems.

References

- [1] Z. Artstein (2007) *Averaging of time-varying differential equations revisited*, Journal of Differential Equations 243.2, pgs. 146–167.
- [2] N. N. Bogolyubov and Y. A. Mitropolsky (1985) *Asymptotic methods in the theory of non-linear oscillations*, Gordon and Breach.
- [3] M. P. Calvo, Ph. Chartier, A. Murua and J. M. Sanz-Serna (2011) *Numerical stroboscopic averaging for ODEs and DAEs*, Applied Numerical Mathematics 61.10, pgs. 1077–1095.
- [4] J. A. Sanders and F. Verhulst (1985) *Averaging methods in nonlinear dynamical systems*, Springer.

Semi-Lagrangian Approach to Kinetic Models in Plasma Simulations

Pietro Nardelli, University of Ferrara

Plasma is a gaseous state of matter consisting of charged particles, specifically electrons and ionised atoms. In modern applied research, several lines of investigation focus on plasma modelling, ranging from nuclear fusion to medicine and astrophysics. In this context, reliable numerical methods are needed to provide accurate simulations. Due to the presence of ions, the motion of plasma particles is governed by electromagnetic forces; moreover, they react to external electric or magnetic fields, changing the overall system behaviour. This complex behaviour makes plasma challenging to simulate efficiently.

In this work, we illustrate one of the methods employed in plasma simulations: the *semi-Lagrangian approach*. The method relies on mathematical tools such as operator splitting [5] and polynomial interpolation [4], and we present it here in the case of the *one-dimensional Vlasov-Poisson system* [1], one of the most studied plasma models. Lastly, we assess the performance of the semi-Lagrangian method with some standard plasma physics test cases, specifically Landau damping and the two-stream instability [3][2].

References

- [1] Chio, Z.C. and Knorr, G. (1976) *The integration of the Vlasov equation in configuration space*, Journal of Computational Physics, Volume 22.
- [2] Mulet, P. and Vecil, F. (2012), *A semi-Lagrangian AMR scheme for 2D transport problems in conservation form*, Journal of Computational Physics.
- [3] Sandberg, R.T., Krasny, R. and Thomas, A.G.R. (2024) *The FARSIGHT Vlasov-Poisson code*, Journal of Computational Physics.
- [4] de Boor, C. (1980), *A Practical Guide to Splines*, McGraw-Hill.
- [5] Strang, G. and MacNavara, S. (2016), *Splitting Methods in Communication, Imaging, Science, and Engineering*, Springer International Publishing Switzerland.

Perturbation of linear non-autonomous parabolic problems in scales of Banach spaces

F. Javier Larcada, Universidad Complutense de Madrid

We present an abstract theory for perturbations of evolution operators of linear non-autonomous parabolic problems on scales of Banach spaces, developing techniques to determine in which spaces of the scale the problem has a solution and where it regularizes. This theory can be applied to specific cases in scales of Bessel potential spaces and (uniform) Lebesgue spaces, addressing perturbations defined by interior potentials, boundary potentials, or transport terms, where the spaces in which there is existence of solution and regularization can be explicitly determined.

This is a joint work with Aníbal Rodríguez-Bernal (UCM).

References

- [1] A. Rodríguez-Bernal (2011) *Perturbation of analytic semigroups in scales of Banach spaces and applications to linear parabolic equations with low regularity data*, SEMA J., 53:3-54.
- [2] J. W. Cholewa and A. Rodríguez-Bernal (2017) *Linear higher order parabolic problems in locally uniform Lebesgue's spaces*, J. Math. Anal. Appl., 449:1-45.
- [3] J. W. Cholewa and A. Rodríguez-Bernal (2025) *On linear Schrödinger parabolic problems in Morrey spaces*, J. Math. Anal. Appl., 554.

Some nonlocal models in population dynamics

A. Casado Sánchez, Universidad de Sevilla

M. Molina Becerra, Universidad de Sevilla

A. Suárez, Universidad de Sevilla

Let $\Omega \subset \mathbb{R}^N$ be a bounded open set, and let $K \in C^0(\overline{\Omega} \times \overline{\Omega}; \mathbb{R}^{M \times M})$. We study a class of nonlocal problems that include an integral term of the form

$$\int_{\Omega} K(x, y)u(y) dy,$$

where the kernel K models the influence of the values of u at distant points. Such operators naturally arise in population dynamics, where $u(x)$ may represent the population density at location x , and the integral term accounts for the individuals moving from one point to another across the habitat.

Our main objective is to establish the existence of nonnegative continuous solutions to such systems using the method of sub- and supersolutions, following an approach similar to that in [1], [2], and [3].

To this end, we construct suitable ordered bounds $\underline{u}$ and $\bar{u}$ and prove the existence of a solution u satisfying $\underline{u} \leq u \leq \bar{u}$. A crucial part of the analysis involves the study of eigenvalues.

Furthermore, we prove a strong maximum principle showing that, under suitable assumptions, any nonnegative solution must be strictly positive unless it vanishes identically. These results extend classical tools from elliptic PDE theory to a nonlocal, vector-valued setting.

Finally, we present numerical simulations that illustrate the applicability of our theoretical results.

References

- [1] F. Li, J. Coville, X. Wang (2017), *On Eigenvalue Problems Arising from Nonlocal Diffusion Models*. Discrete and Continuous Dynamical Systems, Volume 37, Number 2.
- [2] J. Coville (2015), *Nonlocal Refuge Model with a Partial Control*. Discrete and Continuous Dynamical Systems, Volume 35, Number 4.
- [3] J. García-Melián, J. D. Rossi (2009), *A Logistic Equation with Refuge and Nonlocal Diffusion*, Communications on Pure and Applied Analysis, Volume 8, Issue 6.

Dynamic methods in non-autonomous differential equations. Stability, bifurcation, and critical transitions

Paula Martínez Morais, Universidad de Valladolid

This work studies the dynamics of non-autonomous equations from a topological and ergodic perspective. Fundamental classical notions such as exponential dichotomies and their persistence are compiled, as well as concepts of ergodic dynamics applied to non-autonomous systems. Special attention is given to concave flows in finite dimensions, addressing recent results related to stability phenomena, bifurcation, and critical transitions. These models, of particular interest in applied contexts such as mathematical biology or climatology, allow for the capture of relevant nonlinear behaviors in real systems. Additionally, numerical simulations are presented that illustrate the analyzed phenomena and underscore their utility for modeling and qualitative analysis in various branches of applied sciences.

References

- [1] W. A. Coppel (1978). *Dichotomies in Stability Theory*, Springer, Lecture Notes in Mathematics. ISBN: 9780387085364. DOI: <https://doi.org/10.1007/BFb0067780>.
- [2] Peter E. Kloeden and Martin Rasmussen (2011). *Nonautonomous Dynamical Systems*, American Mathematical Society, No. 176.
- [3] Isaac P. Cornfeld, Sergei V. Fomin, and Yakov G. Sinai (2012). *Ergodic Theory*, Springer Science & Business Media, Vol. 245.
- [4] Jesús Dueñas Pamplona et al. (2021). *Métodos dinámicos para el estudio de transiciones críticas en sistemas complejos de la ciencia y la tecnología*.

Control theory in finite-dimensional spaces

First author, Miguel Trujillo Alés.

Tutor bachelor thesis, Diego Araujo de Souza.

The aim of this work is to give an introduction to control and stabilization theory in finite-dimensional spaces. It will be started exposing the basic concepts of controllability in finite-dimensional spaces. Throughout this first part, it is intended to apply these controllability concepts to different types of control systems. First, it will be studied the controllability of linear systems, for both autonomous and non-autonomous cases, with and without control constraints. It will be also seen the controllability of non-linear systems, but only local results. The main idea will be to establish sufficient conditions that ensure the controllability of nonlinear control systems. Finally, it will be also exposed the basic concepts of stability/stabilization in finite-dimensional spaces. Firstly, it is intended to apply these concepts to autonomous linear systems and it will be seen the difficulties that emerge when one tries to stabilize non-autonomous linear systems. It will be also studied the stabilization of non-linear control systems, where the theory of Lyapunov functions will be useful.

References

- [1] E. Trélat, *Contrôle optimal. Théorie & applications. Mathématiques Concrètes. (Francés)* [*Control óptimo. Teoría y aplicaciones. Matemáticas concretas*], Vuibert, Paris, 2005, vi+246 pp.
- [2] E.D. Sontag, *Mathematical control theory. Deterministic finite-dimensional systems. (Inglés)* [*Teoría de control matemático. Sistemas determinísticos finito-dimensionales*] Segunda edición, Textos en Matemáticas Aplicadas, 6, Springer-Verlag, New York, 1998, xvi+531
- [3] J-M. Coron. *Control and nonlinearity. (Inglés)* [*Control y no-linealidad*], Mathematical surveys and monographs, 1956, ISSN 0076-5376 ; v. 136.
- [4] D. Hinrichsen, A. J. Pritchard. *Mathematical Systems Theory I, Modelling, State Space Analysis, Stability and Robustness (Texts in Applied Mathematics). (Inglés)* [*Teoría de sistemas matemáticos I, modelado , análisis de espacio de estdos y robusteza (Textos en matemáticas aplicadas)*]
- [5] M. Bodson. *Explaining the Routh-Hurwitz criterion. A tutorial presentation (Inglés)* [*Explicación del criterio de Routh-Hurwitz. Un tutorial de presentación*], 15 de septiembre de 2019.
- [6] Hassan K.Khalil, *Nonlinear systems. (Inglés)* [*Sistemas no lineales*]. Prentice Hall, 2002.

Additivity of the crossing number under the connected sum of knots

Cristina Gómez Cirera, University of Seville

Knot theory is the branch of Topology studying mathematical knots, which can be intuitively described as a piece of rope that we tie and afterwards we glue its endpoints together. This talk focuses on a long-standing classical Conjecture stating that the crossing number is additive for the connected sum of knots. We introduce the concepts and ideas needed to understand the Conjecture, and present some specific related results. In particular, we show how to prove the Conjecture for two families of knots: alternating knots and torus knots, by following [4] and [2], respectively.

References

- [1] P. R. Cromwell. *Knots and Links*. Cambridge University Press, 2004.
- [2] Y. Diao. “The additivity of crossing numbers”. *Journal of Knot Theory and Its Ramifications* 13.07 (2004), 857-866.
- [3] L. H. Kauffman. “State models and the Jones polynomial”. *Topology* 26.3 (1987), 395-407.
- [4] W. B. R. Lickorish and M. B. Thistlethwaite. “Some links with non-trivial polynomials and their crossing-numbers”. *Commentarii Mathematici Helvetici* 63 (1988), 527-539.

Homotopic Distance: New (Co)homological and Computational Invariants

[Ángel Méndez-Vázquez](#), CITMAga & Universidade de Santiago de Compostela

Enrique Macías-Virgós, CITMAga & Universidade de Santiago de Compostela

David Mosquera-Lois, Universidade de Vigo

The homotopic distance is a unifying notion that generalizes classical concepts from algebraic topology and topological robotics, such as the Lusternik–Schnirelmann category or Farber’s topological complexity. In this talk, we introduce new invariants that provide lower bounds for the homotopic distance: the (co)homological distances, motivated by Fox’s homological category. We study their main properties and illustrate their behavior with representative examples. In particular, we establish results on cup-length and show that our cohomological invariant sharpens previously known bounds in the literature.

We also present a computational approach that reformulates these ideas in the framework of simplicial structures. Furthermore, we develop an algorithm, implemented in SageMath, that allows us to compute this invariant for any triangulable space. This method not only recovers the original invariant but also provides explicit coverings associated with it, a feature that is especially valuable for practical applications.

References

- [1] R. H. Fox. On the Lusternik-Schnirelmann category. *Ann. Math. (2)*, 42, 333–370. 1941.
- [2] E. Macías-Virgós, A. Méndez-Vázquez and D. Mosquera-Lois. A Computational (Co)homological Approach to Contiguity Distance. Preprint, arXiv:2510.18356, 2025.
- [3] E. Macías-Virgós and D. Mosquera-Lois. Homotopic distance between maps. *Math. Proc. Camb. Philos. Soc.*, 172(1):73–93, 2022.

Topology of Complex Polynomials

Manuel García García, Student of the Universitat de València.

In this talk, we are interested in the study of the topology of complex polynomial functions $f : \mathbb{C}^n \rightarrow \mathbb{C}$. The points $c \in \mathbb{C}$ where f fails to be locally a trivial smooth fibration are called *atypical values*. The problem of determining the atypical values of a given polynomial remains open. Among the atypical values, one has the *critical values*, although some atypical values may not be critical. In the literature, such atypical values are often referred to as *critical values at infinity*.

Since 1983, with the pioneering work of Broughton (see [1]), several regularity conditions at infinity have been introduced to ensure the non-existence of critical values at infinity. In this work, we review the most relevant regularity conditions and study their interrelations. In particular, we provide answers to open questions posed by J.J. Nuño-Ballesteros and Lê Dũng Tráng in [2].

References

- [1] S.A. Broughton. *On the topology of Complex Hypersurfaces*. In: *Invent. Math.* 92.2 (1988), pp. 217-241. DOI: <https://doi.org/10.1007/BF01404452>.
- [2] Dũng Tráng Lê and Juan José Nuño Ballesteros. *A remark on the topology of Complex Polynomial Functions*. In: *Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM* 113.4 (2019), pp. 3977-3994. DOI: <https://doi.org/10.1007/s13398-018-0611-z>.

Function approximation with fractal structures

[J. F. Cuevas Rodríguez](#), University of Almeria, Master's student, Almería, 04120, Spain

M. A. Sánchez Granero, University of Almería, Department of Mathematics, Almería, 04120, Spain

J. F. Gálvez Rodríguez, University of Almería, Department of Mathematics, Almería, 04120, Spain

The main goal of this talk is to introduce a framework for approximating and decomposing functions using the idea of fractal structures. By “fractal structures,” I mean families of coverings that are recursive and topologically refined. They provide a natural way to study mathematical objects that are highly complex or irregular. One of the key advantages of this approach is that it lets us define approximation processes in very general topological spaces, without having to rely on classical orthogonal bases like polynomials or sine and cosine functions.

In the first part, I'll explain how to build the theoretical foundations for equipping a topological space with a fractal structure. Along the way, I'll highlight some important properties, such as irreducibility, the starbase condition, and the level separation property, or LSP. I'll also touch on some broader notions, like prefractal structures and GF-maps, which give extra flexibility to the framework.

After that, I'll introduce the central ideas of approximation functions and decomposition functions. The main takeaway is that, under suitable conditions, just by choosing functions at each level of a fractal structure, we can recover any continuous function on a compact space.

To make things a bit more concrete, I'll briefly show how some classical results in topology can be rephrased in terms of fractal structures. This isn't the main focus, but it helps to see that the framework connects naturally with well-known theorems, and that it isn't an isolated construction.

Finally, I'll move on to applications. I'll present two examples where fractal structures prove especially useful. The first is in the modeling of investment strategies, specifically Pairs Trading, where the multiscale nature of fractals helps to detect relationships between financial assets. The second is in the rendering of coastal maps, where fractal structures are very effective at describing irregular boundaries with great precision, such as those found in geographic territories.

References

- [1] F. G. Arenas, M. A. Sánchez-Granero. Hahn-Mazurkiewicz revisited: A new proof. *Houston J. Math.*, **28**(4):753–769, 2002.

- [2] M. Fernández-Martínez, M. A. Sánchez-Granero. Fractal dimension for fractal structures. *Topology Appl.*, **163**:93–111, 2014.
- [3] M. Fernández-Martínez, M. A. Sánchez-Granero. A new fractal dimension for curves based on fractal structures. *Topology Appl.*, **203**:108–124, 2016.
- [4] B. Mandelbrot. How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science*, **156(3775)**:636–638, 1967. <http://www.jstor.org/stable/1721427>

Cohomology basis graphs on right-angled Artin groups

Daniel Gómez Gutiérrez, University of Seville

Cohomology basis graphs is a recent tool that mimics the behavior of the Artin presentation of right-angled Artin groups (also known as RAAGs) with other given presentations, in order to study RAAGs when the Artin presentation is unknown and to hopefully retrieve the original graph. We share advancements on the properties of these graphs related to the problem of obtaining the Artin presentation from them.

References

- [1] Thomas Koberda. *Geometry and combinatorics via right-angled Artin groups*. 2021. arXiv: 2103.09342 [math.GR]. URL: <https://arxiv.org/abs/2103.09342>.
- [2] Ramón Flores, Delaram Kahrobaei, Thomas Koberda, and Corentin Le Coz. *Right-angled Artin groups and the cohomology basis graph*. 2024. arXiv: 2309.05495 [math.GR]. URL: <https://arxiv.org/abs/2309.05495>.

Spherical-type Artin-Tits groups: generalizing curves in the n -punctured disc

Pablo Regalado García, University of Seville

Braids are one of the most studied groups within a much larger family, called Artin-Tits groups, in the field of Geometric Group Theory. Its topological viewpoint as a mapping class group allows us to obtain powerful results that can be conjectured to be true in a more general setting, where the topology is no longer a tool for studying this family of groups.

In this talk, we will give an approach to Nielsen-Thurston classification theorem for mapping class groups and will focus on spherical-type Artin-Tits groups. We will discuss how topological ideas from this classification theorem can be generalized into purely algebraic objects and which definitions remain to be “translated” into algebra, focusing on the Ribbon Conjecture for Artin-Tits groups and its relation to the topic.

References

- [1] María Cumplido (2022) *The Conjugacy Stability Problem for Parabolic Subgroups in Artin Groups*, Mediterranean Journal of Mathematics, volume 19, article number 237.
- [2] Luis Paris (1997) *Parabolic Subgroups of Artin Groups*, Journal of Algebra, volume 196, pages 369-399.
- [3] Benson Farb, Dan Margalit (2012) *A Primer on Mapping Class Groups*, Princeton University Press.
- [4] María Cumplido, Volker Gebhardt, Juan González-Meneses, Bert Wiest (2019) *On parabolic subgroups of Artin-Tits groups of spherical type*, Advances in Mathematics, volume 352, pages 572-610.

Algebraic fibering of the pure symmetric automorphism group of a RAAG

Marcos Escartín Ferrer, Universidad de Zaragoza

The family of right-angled Artin groups (RAAGs) plays a central role in geometric and combinatorial group theory. Given a simplicial graph, the associated RAAG is defined by assigning one generator to each vertex, with commutation relations determined by the edges of the graph.

Within the automorphism group of a RAAG, the pure symmetric automorphism group is the subgroup generated by the partial conjugations. A group G is said to fibre if there exists an epimorphism $G \rightarrow \mathbb{Z}$ whose kernel is finitely generated.

In this talk, I will describe a complete characterization of the fibering properties of pure symmetric automorphism groups of RAAGs in terms of combinatorial features of the defining graph. I will also explain how these properties are connected to the vanishing of the first ℓ^2 -Betti number.

Commensurators and Grigorchuk groups

Clara Skowronek Santos, Universidad Complutense de Madrid

Lately the class of groups which are totally disconnected, locally compact, simple and compactly generated has gain importance. Some of the examples of groups in this class that are already known are obtained as commensurators of profinite branch groups. In this talk I will discuss the commensurators of two such groups, namely two of the Grigorchuk groups. These groups are finitely generated groups of rooted tree automorphisms, and their commensurators are a generalization of their automorphism groups. I will describe the branch structure of the two aforementioned groups, and I will introduce some key results about commensurators—most notably, that there is an isomorphism between the abstract commensurator of certain groups and their relative commensurators in the homeomorphism group of the boundary of the tree, and that the commensurators of certain groups contain a Higman-Thompson group. Finally, I bring together these results to compute the commensurator of the two groups under consideration.

References

- [1] R. I. Grigorchuk (1985). Degrees of growth of finitely generated groups, and the theory of invariant means. *Mathematics of the USSR-Izvestiya*, 25(2):259.
- [2] R. I. Grigorchuk (2000). Just infinite branch groups. In M. du Sautoy, D. Segal, and A. Shalev (eds.), *New Horizons in pro-p Groups*, pp. 121–179. Birkhäuser, Boston, MA.
- [3] C. E. Röver (2002). Abstract commensurators of groups acting on rooted trees. *Geometriae Dedicata*, 94(1):45–61. Springer.

Tobit filtering for nonlinear networked systems: A symmetric encryption framework

Shuo Yang, Harbin University of Science and Technology, China; University of Jaén, Spain

Jun Hu, Harbin University of Science and Technology, China

Raquel Caballero-Águila, University of Jaén, Spain

Antonia Oya-Lechuga, University of Jaén, Spain

In recent decades, the filtering problem has garnered substantial attention owing to its broad application prospects, which have given rise to numerous filtering schemes developed for diverse performance requirements. Notably, real engineering systems are normally characterized by inherent nonlinearities [1], and thus significant research efforts have been directed toward the design of filtering algorithms for nonlinear systems. Furthermore, most studies assume that the measurement outputs are transmitted under perfect conditions from the sensors, which is unrealistic in practice, especially for low-cost devices. Intrinsic physical limitations typically lead to censored measurements, which have become an increasingly key focus in filtering research [2]. On the other hand, the rapid development of network technology has resulted in information security being a critical research topic [3]. In particular, eavesdroppers can infer sensitive system information by intercepting the measurement data transmitted over a shared channel, thereby posing a serious security risk. Motivated by the preceding discussions, the focus of this research is to propose a novel symmetric encryption-based Tobit filtering algorithm to ensure that the upper bound of the filtering error covariance is minimized at each time step, and the corresponding filter gain is derived. Additionally, a sufficient condition is provided such that the filtering error is uniformly bounded in the mean-square sense to evaluate the performance of the presented filtering approach. Finally, the effectiveness of the developed Tobit filtering scheme is illustrated through a simulation experiment.

References

- [1] B. Jiang, H. Dong, Z. Gao, Y. Shen, and F. Yang, Linear-fitting-based recursive filtering for nonlinear systems under encoding-decoding mechanism, *Sci. China-Inf. Sci.*, 67, 152203, 2024.
- [2] Y. Li and J. Liang, Distributed Tobit Kalman filtering for random parameter 2-D system: Dealing with amplify-and-forward relay and stochastic communication protocol, *Inf. Sci.*, 718, 122355, 2025.
- [3] F. Tao and D. Ye, Secure state estimation against eavesdropping attacks based on time-varying coding and noise-adding, *IEEE Trans. Netw. Sci. Eng.*, 11(1), 174–184, 2024.

Morphisms of rewriting systems

[Raúl Ruiz Mora](#), Universitat de València

Enric Cosme Llópez, Universitat de València

Universal algebra studies algebras relative to a signature and their properties without specifying the nature of the structures. In particular, to relate algebras with different signatures, the concept of a derivor is useful, as it allows interpreting operation symbols from the source signature as derived operation symbols in the target signature. Following this construction we can define morphisms between rewriting systems interpreting rewriting rules from the source rewriting system as derived paths in the target rewriting system. In this presentation, we introduce the concept of derivor and use it as a foundation to construct morphisms between higher-order rewriting systems.

References

- [1] J. Climent Vidal and E. Cosme Llópez. *From higher-order rewriting systems to higher-order categorial algebras and higher-order Curry-Howard isomorphisms*, 2024. [arXiv:2402.12051](#).

Solutions to Nonlinear Differential Systems with Nontrivial Components via Fixed Point Theory

Laura María Fernández Pardo, Departamento de Estadística, Análise Matemática e Optimización, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain.

The main objective of this talk is to present sufficient conditions for the existence of solutions to differential systems with nonlinearities having nontrivial components, using various vectorial versions of Krasnosel'skiĭ's compression-expansion fixed point theorem in cones. The key idea behind these methods is to transform the existence problem into a fixed point one.

The vectorial versions we employ apply to compact operators of the form $T = (T_1, T_2)$ acting on the product of normed spaces $X \times Y$. Some of these versions are presented in [1, 2], where conditions are imposed separately on each component. In addition, we will use a novel version explicitly formulated in terms of norms.

As an advantage of these vectorial versions we can emphasize that they guarantee the existence of a fixed point $(x_1, x_2) \in X \times Y$ such that $\|x_j\| > 0$ for each $j \in \{1, 2\}$, which, by identifying solutions as fixed points of a certain operator, ensures both components are nontrivial (something not guaranteed by the classical Krasnosel'skiĭ's fixed point theorem [3]). Additionally, they allow applications to systems where one component exhibits expansive behavior while the other is compressive, in contrast to the classical case.

References

- [1] R. Precup, A vector version of Krasnosel'skiĭ's fixed point theorem in cones and positive periodic solutions of nonlinear systems, *J. Fixed Point Theory Appl.* **2** (2007), 141–151.
- [2] J. Rodríguez-López, *A fixed point index approach to Krasnosel'skiĭ-Precup fixed point theorem in cones and applications*, *Nonlinear Analysis*, **226** (2023), No. 113138.
- [3] M.A. Krasnosel'skiĭ's, *Fixed points of cone-compressing or cone-extending operators*, *Soviet Mathematics*, **135** (1960), 527-530.

Periodic solutions of singular differential equations: from the first result onward

María P. Armesto, Universidade de Santiago de Compostela

Singular differential equations have sparked the attention of several researchers due to their appearance in problems with gravitational forces, electricity, intermolecular dynamics, and so on. That singularity — a point in the domain where the function defining the differential equation tends to infinity — often arises when modeling such phenomena. A compilation of real-world examples where singular differential equations show up can be found in [3].

Both the Dirichlet and Neumann problems for singular differential equations have been treated by many authors, and a multitude of existence and uniqueness theorems regarding these boundary value problems were proved early in the literature. However, it was not until 1987 that the first result concerning the periodic boundary value problem was stated. These results, given by Lazer and Solimini [2], provided the existence of periodic solutions — via the method of upper and lower solutions — of the class of equations

$$x'' \pm g(x) = h(t), \tag{1}$$

known as the Lazer-Solimini equations, where the singular function g is inspired by terms of the form $\pm 1/x^p$, $p > 0$. In this talk, we will review their proofs for both the attractive and repulsive cases — with a positive or a negative singular term, respectively.

Following the work by Lazer and Solimini, some authors have been interested in its possible generalizations, while others have studied different singular periodic problems. This talk focuses on some extensions of Lazer and Solimini's work: we will introduce, on the one hand, some results weakening the regularity conditions on g and h in equation (1) and, on the other hand, some results on the existence of bouncing solutions — non-classical solutions allowed to touch the singularity — for equation (1). Some of these results can be found in [1].

References

- [1] Patrick Habets, Luis Sanchez (1990) *Periodic solutions of some Liénard equations with singularities*, Proceedings of the American Mathematical Society, 104(4), pp. 1035–1044.
- [2] Alan C. Lazer, Sergio Solimini (1987) *On periodic solutions of nonlinear differential equations with singularities*, Proceedings of the American Mathematical Society, 99(1), pp. 109–114.
- [3] Pedro J. Torres (2015) *Mathematical models with singularities: a zoo of singular creatures*, Atlantis Briefs in Differential Equations, 1, Atlantis Press, Paris, 1st ed.

Using the Frobenius Number to Determine the Transient of non-linear boolean dynamical systems.

[Mario Jacobo Motiño Palma](#), Mathematical Sciences Dept. Univ. of Puerto Rico, Mayaguez.

Arnaldo José Vera López, Mathematical Sciences Dept. Univ. of Puerto Rico, Mayaguez.

Eiver Rodríguez Pérez, Mathematical Sciences Dept. Univ. of Puerto Rico, Mayaguez.

Omar Colón-Reyes, Mathematical Sciences Dept. Univ. of Puerto Rico, Mayaguez.

Monomial Boolean discrete dynamical systems are mathematical functions used to model discrete interactions in various contexts, such as networks and biology. Each such system can be associated with a dependency graph, which describes how the variables are related to one another. A key problem is, given a fixed-point discrete dynamical system f , to determine the smallest positive integer t for which $f^t = f$, that is, when the system stops changing state after applying its dynamics t times. This number t is known as the transient of the system. Normally, computing this transient requires testing all possible states of the system, which can be complicated when many states exist.

The goal is, to directly relate the structure of the system (in particular, its dependency graph) to its transient, so as to avoid exhaustive computation. We study a family of systems whose dependency graph is formed by the union of three connected cycles of prime lengths. Surprisingly, in certain cases, computing the transient is equivalent to finding the *Frobenius number* for three integers. By taking advantage of results by Tripathi, see [1], we were able to obtain an explicit formula for determining the transient of a particular type of systems. Our work offers a much simpler and more direct way to compute the transient from the structure of the dependency graph.

References

- [1] Amitabha Tripathi, Formulae for the Frobenius number in three variables, Journal of Number Theory, 170, 2017.

Bohnenblust–Hille inequality and applications in learning theory

Isabel María Moreno Cuadrado, Complutense University of Madrid

Boolean functions, defined on the hypercube $\{-1, 1\}^n$, play a central role in many areas of mathematics and theoretical computer science. However, in many relevant situations, the governing function is unknown: we do not have direct access to it, but only to a limited number of examples. This raises the fundamental problem of how to reconstruct or approximate an unknown function from partial information. This is precisely one of the central questions addressed in learning theory.

In this talk, we will discuss some analytical tools used to tackle this problem, such as hypercontractivity and Blei’s inequality. Building on these, we establish a discrete version of the Bohnenblust–Hille inequality, a classical result in the analysis of Dirichlet series which, in this context, provides precise control over the $\ell^{\frac{2d}{d+1}}$ norm of the Fourier coefficients of any function defined on the cube, and hence over its spectral behavior.

Finally, we explore possible connections with open problems in quantum information theory; in particular, with the Aaronson–Ambainis conjecture, a deep proposal that sets fundamental limits on quantum advantage.

Carleson's Theorem

Miguel Rodríguez Alegre, Universitat de Valencia

How could a Fourier series converge to its original function? Many convergence results are demonstrated easily with Fourier analysis tools, such as uniform convergence, or convergence in the $L^2([0, 1])$ -space norm. But what about pointwise convergence? Under certain assumptions, such as almost everywhere continuity for example, it is not hard to show that the Fourier series converges pointwise almost everywhere to the original function. Nevertheless, pointwise convergence with $L^p([0, 1])$ functions is much harder to handle. Carleson's celebrated theorem (1966) establishes this result 50 years after its conjecture. Its proof requires some innovative techniques that became influential during the next decades, even though they do not allow us to demonstrate problems that remain open today.

References

- [1] J. DUOANDIKOETXEA, *Fourier Analysis*. Graduate Studies in Mathematics, Volume 29. American Mathematical Society. 2001.
- [2] L. GRAFAKOS, *Classical Fourier Analysis, Third edition*. Springer. Undergraduate Texts in Mathematics. 2014.
- [3] L. GRAFAKOS, *Modern Fourier Analysis, Third edition*. Springer. Undergraduate Texts in Mathematics. 2014.
- [4] J. HICKMAN, *Fourier and Harmonic Analysis. Lecture 6: Maximal functions and almost everywhere convergence*. University of Edinburgh. 2021.
- [5] T. HYTÖNEN, *Time-Frequency Analysis*. Lecture notes of a course at the University of Helsinki. 2012.
- [6] M. LACEY, *Carleson's Theorem: Proof, Complements, Variations* Publicacions Matemàtiques, Volumen 48, Número 2 (2004).
- [7] V. LIE, *The pointwise convergence of Fourier Series (I). On a conjecture of Konyagin* Journal of the European Mathematical Society, Volume 19, n^o6 (2017).
- [8] W. SCHLAG, C. MUSCALU, *Classical and multilinear harmonic analysis. Volume 1.* Cambridge University Press. 2013.

Spinors, Möbius maps and spheres close to lightspeed

Alonso Diaz Uña, University of Seville

In special relativity, space-time events are elements of $\mathbb{R}^4$ whose coordinates (x, y, z, t) are related by linear transformations from the Lorentz group $\mathcal{L}$. One prediction is a length contraction that all moving objects undergo in the direction of their movement. However, their visual appearance is also distorted by optic illusions due to the finite speed of light. In [2], mathematical physicist Roger Penrose related the problem of determining this final appearance with machinery from group theory and complex analysis. A modern treatment of this argument can be found in [1]

We will identify our field of vision with a 2-sphere $F \subseteq \mathbb{R}^4$, and the outline of an object as a curve $\mathcal{C}$ on it. We will introduce the 2-to-1 map $\text{Spin} : \text{Sl}(2, \mathbb{C}) \rightarrow \mathcal{L}$, which will give us an isomorphism $\mathcal{L}$ with $\text{PSl}(2, \mathbb{C})$, the group of Möbius transformations. For a given Lorentz map Λ in $\mathbb{R}^4$ relating observer and object, we will demonstrate that its action on F (where we do not mean $\Lambda|_F$, but the one induced from the effect of Λ on 'light rays' intersecting F) will coincide with the associated Möbius transformation f_Λ on $\hat{\mathbb{C}} \cong S^2$, so that the transformed outline $\mathcal{C}'$ of the moving object can be analyzed from this fact. In particular, the property of Möbius maps of preserving (generalized) circles will imply that spheres, however fast, always show a circular outline, even if their surface appears contorted, making length contraction invisible in their case.

At last, we will discuss briefly the Terrell rotation effect reviewed in [2] and a recent experimental realisation of this phenomenon, [4], that motivated this talk. The slides will be expanded with geometrical visualizations done with Manim software.

References

- [1] Naber, G. L. (2011). The Geometry of Minkowski Spacetime. En Applied mathematical sciences. <https://doi.org/10.1007/978-1-4419-7838-7>
- [2] Penrose, R. (1959). The apparent shape of a relativistically moving sphere. Mathematical Proceedings Of The Cambridge Philosophical Society, 55(1), 137-139. <https://doi.org/10.1017/s0305004100033776>
- [3] Terrell, J. (1959). Invisibility of the Lorentz Contraction. Physical Review, 116(4), 1041-1045. <https://doi.org/10.1103/physrev.116.1041>
- [4] Hornof, D., Helm, V., De Dios Rodriguez, E., Juffmann, T., Haslinger, P., Schattschneider, P. (2025). A snapshot of relativistic motion: visualizing the Terrell-Penrose effect. Communications Physics, 8(1). <https://doi.org/10.1038/s42005-025-02003-6>

Discontinuous Galerkin approach for isogeometric methods

Ángel Pita-da-Veiga, Departamento de Matemática Aplicada, Universidade de Santiago de Compostela, 15782, Santiago de Compostela, España

Jerónimo Rodríguez, Departamento de Matemática Aplicada, Universidade de Santiago de Compostela, 15782, Santiago de Compostela, España

Rafael Vázquez, Departamento de Matemática Aplicada, Universidade de Santiago de Compostela, 15782, Santiago de Compostela, España

Isogeometric methods, based on spline functions, have emerged as a variation of the finite element method, with the goal of integrating the geometry of the computational domain directly into the basis functions used to discretize the underlying functional space. This approach facilitates the preservation of the de Rham diagram, which plays a fundamental role in certain physics-based problems, such as those in electromagnetism. In this work, we consider two sequences of de Rham complexes: the primal sequence, starting from the tensor-product space of polynomials of degree p , and the dual sequence, beginning with polynomials of degree $p - 1$, to ensure dimensional compatibility across the involved spaces.

Building on the fast method introduced in [1], we explore the extension of this formulation to time-dependent problems involving multiple subdomains. We aim to obtain this generalization through discontinuous Galerkin techniques (see [2]) and frameworks inspired by Friedrichs systems (see [3]). We present the derivation of the corresponding variational formulation and provide numerical results illustrating the convergence properties of the method.

References

- [1] Kapidani, B., & Vázquez, R. (2023). *High order geometric methods with splines: Fast solution with explicit time-stepping for Maxwell equations*. *Journal of Computational Physics*, **493**, 112440. <https://doi.org/10.1016/j.jcp.2023.112440>
- [2] Arnold, D. N., Brezzi, F., Cockburn, B., & Marini, L. D. (2002). *Unified analysis of discontinuous Galerkin methods for elliptic problems*. *SIAM Journal on Numerical Analysis*, **39**(5), 1749–1779. <https://doi.org/10.1137/S0036142901384162>
- [3] Imperiale, S., Joly, P., & Rodríguez, J. (2025). *Stability of time stepping methods for discontinuous Galerkin discretizations of Friedrichs' systems* [Preprint]. HAL. <https://hal.science/hal-05010903>

Lipschitz Extensions in Fuzzy Metric Spaces: A Generalization of the McShane–Whitney Theorem

[Aurora Sánchez Martín-Orozco](#), Universitat de València

Jesús Rodríguez López, Universitat Politècnica de València

Enrique A. Sánchez Pérez, Universitat Politècnica de València

A classical problem in mathematics concerns the extension of functions defined on a subset of a space to the whole space while preserving certain properties. In this work, we focus on the McShane–Whitney theorem, which guarantees the extension of Lipschitz functions on metric spaces without increasing their Lipschitz constant, and we explore its generalization to the framework of fuzzy metric spaces, a setting designed to model imprecise data and uncertainty.

Building on the construction of Jiménez-Fernández et al. (2021) [4], we develop a generalized version of the McShane–Whitney extension theorem in the fuzzy context. We illustrate the theoretical results with explicit examples and propose a practical application: a fuzzy Lipschitz regression model for predicting fish catches in Mediterranean harbors using real data.

This study aims to bridge the gap between the theoretical development of Lipschitz extensions in fuzzy settings and their potential applications to real-world problems involving uncertainty.

References

- [1] S. Cobzas, R. Miculescu, and A. Nicolae. *Lipschitz Functions*, volume 2241 of *Lecture Notes in Mathematics*. Springer International Publishing, 2019.
- [2] Instituto Español de Oceanografía (IEO-CSIC). Estadísticas de capturas anuales por puerto (2009–2021) de pequeños pelágicos en el litoral levantino. Comunicación personal, 2025. Datos facilitados por el Centro Oceanográfico de Murcia a través de la colaboración con el Prof. José M. Bellido.
- [3] A. George and P. Veeramani. On some results in fuzzy metric spaces. *Fuzzy Sets and Systems*, 64(3):395–399, 1994.
- [4] E. Jiménez-Fernández, J. Rodríguez-López, and E.A. Sánchez-Pérez. McShane–Whitney extensions for fuzzy Lipschitz maps. *Fuzzy Sets and Systems*, 406:66–81, 2021.

Communication without Distortion: Soliton Propagation in Nonlinear Media

[Fernando Carreño-Navas](#), IMUS, Universidad de Sevilla.

Renato Alvarez-Nodarse, IMUS and Departamento de Análisis Matemático, Universidad de Sevilla

Niurka R. Quintero, Departamento de Física Aplicada I, Escuela Técnica Superior de Ingeniería Informática, Universidad de Sevilla.

Reliable communication requires the transmission of signals without distortion or loss of information as they propagate through real media, which are inherently nonlinear, dispersive, and dissipative.

A promising mechanism to achieve this is the use of solitons, localized wave packets that preserve their shape and energy balance even in complex environments. In this work, we address the question: *under what conditions can two interlocutors establish a stable communication channel without information loss?* To answer this, we study a modified Nonlinear Schrödinger (NLS) equation,

$$i\phi_t + \phi_{xx} + 2|\phi|^{2\kappa}\phi = re^{2it}\phi^* - i\rho\phi,$$

which incorporates a dissipation term ($-i\rho\phi$) and a parametric forcing term ($re^{2it}\phi^*$) that compensates for the energy loss. To model a broader class of media, we also introduce the parameter κ , which modifies the nonlinearity of the system.

We focus on the existence and stability of two types of solitons, ϕ_+ and ϕ_- , which differ only in their relative phase with respect to the external force, but exhibit drastically different propagation behavior. By combining analytical techniques, through the Sturm-Liouville problem associated with the linearized equation, and numerical simulations, we determine the stability regions in parameter space (r, ρ, κ) where solitons can exist and reliably carry information. These results provide a theoretical framework about how information can be transmitted in realistic physical media without degradation.

References

- [1] Carreño-Navas, F., Alvarez-Nodarse, R., & Quintero, N. R. *Oscillatory instability and stability of stationary solutions in the parametrically driven, damped nonlinear Schrödinger equation*. Physica D Nonlinear Phenomena, 134611, (2025).
- [2] I. V. Barashenkov, M. M. Bogdan, and V. I. Korobov. *Stability Diagram of the Phase-Locked Solitons in the Parametrically Driven, Damped Nonlinear Schrödinger Equation*. Europhys. Lett., 15:113, (1991).

A Brief Introduction to Computer-Assisted Techniques in Analysis

Miguel M.G. Pascual-Caballo, Instituto de Ciencias Matemáticas

Computer-assisted proofs are, and will continue to be, a powerful tool for establishing results that, for now, we do not know how to prove otherwise. In analysis, the mathematical tool underlying these methods is interval arithmetic. In this talk, we will present its basic principles as well as some of the types of arguments that can be carried out with it.

References

- [1] Javier Gómez-Serrano (2018) Computer-assisted proofs in PDE: a survey, <https://arxiv.org/abs/1810.00745>

Global Regularity for the 2D Navier–Stokes Free Boundary Problem

[Paula Luna-Velasco](#), Universidad de Sevilla-Universidad de Granada.

Francisco Gancedo, Universidad de Sevilla

Eduardo García-Juárez, Universidad de Sevilla

The Navier–Stokes equations describe the motion of incompressible fluids and remain a central source of challenging mathematical problems. In this talk, I will present new results on the two-dimensional free boundary problem for two immiscible fluids. We prove global-in-time well-posedness in the setting of nonnegative density and establish the persistence of the natural $C^{1+\gamma}$ regularity of the interface under sharp assumptions on the initial velocity. In addition, we extend the analysis to the case of density-dependent viscosity, showing global regularity when the viscosity contrast is small.

References

- [1] F. Gancedo, E. García-Juarez, P. Luna-Velasco (2025) *On 2D Navier-Stokes free boundary: nonnegative density and small viscosity contrast*. Preprint arXiv:2507.09333.

The phenomenon of quenching in a system with non-local diffusion

[Sergio Junquera](#), Universidad Complutense de Madrid

José M. Arrieta, Universidad Complutense de Madrid, ICMAT

Raúl Ferreira, Universidad Complutense de Madrid

Diffusion models appear in multiple sciences such as biology, physics or even economics. They come up naturally as a broad class of natural processes, like the transport of matter due to random molecular motions. The most common diffusion operator in dynamical systems is the laplacian Δ , which is derived from Fick's laws and leads us to the local diffusion model $u_t = \Delta u$. It is called local diffusion because this model only considers the possibility of the particles moving very short distances in each instant of time. However, there can be more than that. There are other phenomena, such as the propagation of a pathogen, in which the particles could jump long distances in each instant of time thanks to various means of transport. We call this non-local diffusion and it is modeled by different operators, such as those of the type $J * u - u$, where the kernel J is a density function of the probability of any jump happening, see [AMRT]. The most famous non-local operator is the fractional laplacian $(-\Delta)^s$, which has by itself spawned a whole array of literature such as [P].

The phenomenon of quenching in a dynamical system consists of the explosion of the velocity of the solution while the solution itself remains bounded. It was first assessed by Hideo Kawarada in 1974 for the equation $u_t = u_{xx} + (1-u)^{-1}$, where it happens whenever the solution reaches the value $u = 1$, see [K]. The phenomenon of quenching appears naturally in physical models such as the nonlinear heat conduction in solid hydrogen, see [R], or the Arrhenius Law in combustion theory, see [CK]. It is for this reason that quenching has been the subject of numerous studies since Kawarada's paper, as seen in the surveys [C, L1].

The aim of this talk is to speak about our study of a system of equations with weakly coupled singular absorption terms and a non-local diffusion operator of the type convolution with a smooth kernel and the quenching phenomena that arises. First we will offer a suitable introduction to the non-local diffusion operator and the quenching phenomenon so that the talk can be followed by anyone interested but without prior knowledge on these topics. Then we will show our results about the system, which tackle the appearance of stationary solutions, the quenching rates of both components, the possibility of both components presenting quenching at the same time and the added difficulties this problem presents with respect to the single equation with non-local diffusion studied in [Fe1].

Acknowledgements: The Authors acknowledge financial support received from FPU21 grant from Ministry of Science, Innovation and Universities of Spain.

References

- [AMRT] F. Andreu, J. M. Mazón, J. D. Rossi and J. Toledo. *Non-local Diffusion Problems*. Mathematical Surveys and Monographs, 165, Amer. Math. Soc., Providence, RI, 2010 R.

- Soc. Mat. Esp., Madrid, 2010; MR2722295
- [C] C. Y. Chan. *Recent advances in quenching phenomena*. Proc. Dynam. Systems. Appl. **2** (1996), 107–113.
- [CK] C. Y. Chan, P. C. Kong. *A Thermal Explosion Model*. Applied Math. and Computation. **71** (1995), 201–210.
- [Fe1] R. Ferreira. *Quenching Phenomena for a Non-Local Diffusion Equation with a Singular Absorption*. Israel Journal of Mathematics **184**, 387–402, 2011
- [K] H. Kawaiada. *On solutions of initial-boundary problem for $u_t = u_{xx} + 1/(1 - u)$* . Publ. Res. Inst. Math. Sci. **10** (1974/75), 729–736.
- [L1] H. A. Levine. *The phenomenon of quenching: a survey*. In “Trends in the Theory and Practice of Nonlinear Analysis”, (V. Lakshmikantham, ed.), Elsevier Science Publ., North Holland, 1985, pp. 275–286.
- [P] C. Pozrikidis, *The fractional Laplacian*, CRC Press, Boca Raton, FL, 2016; MR3470013
- [R] G. Rosen. *Nonlinear heat conduction in solid H_2* . Phys. Review B, **19** (1979), 2398–2399.

Revisiting Hadwiger's Theorem: A more accessible proof

Jorge Santiago Ibáñez Marcos, Universidad Complutense de Madrid (UCM)

Pedro Tradacete, ICMAT

Ignacio Villanueva, UCM

Let $\mathcal{K}^n$ denote the set of convex, compact sets of $\mathbb{R}^n$. We say that a mapping $V : \mathcal{K}^n \rightarrow \mathbb{R}$ is a valuation if it satisfies the additivity property:

$$V(K) + V(L) = V(K \cup L) + V(K \cap L)$$

whenever $K, L, K \cup L \in \mathcal{K}^n$. Hadwiger's classical theorem states that any continuous, translation-invariant, and n -homogeneous valuation on $\mathcal{K}^n$ must be a multiple of the volume. This result is a cornerstone of the theory of convex geometry and valuation theory with broad applications [1].

The theorem has traditionally been proved using tools from measure theory and the theory of valuations. In this talk, we present a new proof based on a counting argument and on the automatic continuity theory, a powerful framework for analyzing the continuity of algebraic structures. By applying these tools, we are able to provide a more direct proof that avoids some of the more advanced machinery typically involved in the traditional approach. Our method also leads to a more accessible understanding of Hadwiger's theorem, requiring less background in measure theory or valuation theory. Furthermore, our proof allows us to extend the theorem by showing that the volume is the unique valuation satisfying weaker conditions.

References

- [1] Rolf Schneider (2014) *Convex bodies: The Brunn-Minkowski Theory*, Cambridge University Press, Cambridge, 2nd ed.

A discrete approach to functional Zhang's inequality

Julia Sánchez-Loscertales, Universidad de Zaragoza

Zhang's inequality states that among all convex bodies $K \subseteq \mathbb{R}^n$, the affine invariant quantity $|K|^{n-1}|\Pi^*K|$, where Π^*K is the polar projection body of K and $|\cdot|$ denotes the Lebesgue measure, is minimized when K is a simplex.

In 1998, Gardner and Zhang [5] gave a new proof of Zhang's inequality, using an extension of Berwald's inequality [4], which is a reverse Hölder inequality for concave functions.

In 2020, Alonso-Gutiérrez, Bernués and González Merino [1, 2] proved a functional version of Zhang's inequality for log-concave functions and Alonso-Gutiérrez, Lucas and Martín Goñi [3] obtained a discrete version (for the lattice point enumerator measure) of Zhang's inequality.

In this talk, we will adapt the previous results to the discrete setting in the functional context.

References

- [1] D. ALONSO-GUTIÉRREZ, J. BERNUÉS, B. GONZÁLEZ MERINO. *Zhang's inequality for log-concave functions*. Geometric Aspects of Functional Analysis - Israel Seminar (GAFA) 2017–2019. Lecture Notes in Mathematics **2256** (2020), 29–48.
- [2] D. ALONSO-GUTIÉRREZ, J. BERNUÉS, B. GONZÁLEZ MERINO. *An extension of Berwald's inequality and its relation to Zhang's inequality*. Journal of Mathematical Analysis and Applications **486** (1) (2020), 123875.
- [3] D. ALONSO-GUTIÉRREZ, E. LUCAS, J. MARTÍN GOÑI. *A discrete approach to Zhang's projection inequality*. arXiv preprint arXiv:2509.14986 (2025).
- [4] L. BERWALD. *Verallgemeinerung eines Mittelwertsatzes von J. Favard, für positive konkave Funktionen*. Acta Mathematica **79** (1947), 17–37.
- [5] R. J. GARDNER, G. ZHANG. *Affine inequalities and radial mean bodies*. American Journal of Mathematics **120** (3) (1998), 505–528.

Some topics on the L_p Brunn Minkowski Theory

Matheus Resende Guedes, Universidad de Sevilla

The Brunn–Minkowski theory lies at the heart of convex geometry, connecting volume, set addition, and affine invariants. Its L_p -extension, initiated by Firey [1], generalizes the Minkowski sum by combining support functions by their p -mean, has led to a rich field of discoveries, including L_p analogues of the Busemann–Petty centroid inequality and the Petty projection inequality [2]. The aim of this talk is based on our work over the past year exploring some of these ideas, in particular L_p versions of classical operators such as centroid and projection bodies, as well as generalizations of the covariogram function.

References

- [1] W. J. Firey, “ p -Means of convex bodies,” *Mathematica Scandinavica*, vol. 10, pp. 17–24, 1962.
- [2] E. Lutwak, D. Yang, and G. Zhang, “ L_p affine isoperimetric inequalities,” *Journal of Differential Geometry*, vol. 56, no. 1, pp. 111–132, 2000.

Loomis-Whitney type inequalities

Beatriz Marín Gimeno, Universidad de Murcia

Given $K \subseteq \mathbb{R}^n$, a convex body (a convex and compact set with non-empty interior), we can bound its volume by the volume of the box whose edges have lengths equal to the volumes of the projections of K onto the coordinate axes. Using this idea, in 1949, Loomis and Whitney proved, by approximating sets with boxes, that

$$|K|^{n-1} \leq \prod_{i=1}^n |P_{e_i^\perp} K|$$

where $|\cdot|$ denotes volume, and $P_{e_i^\perp} K$ denotes the projection of K onto $\langle e_i \rangle^\perp$. Later, Bollobás and Thomason generalized the previous result to the volumes of projections of convex bodies onto other subspaces. Dual results to the above provide lower bounds for the volume of K , in which projections are replaced by sections.

The aim of this talk is to present the classical results known of this type, as well as more recent ones that are considered local in nature for projections, and certain results that improve upon the existing ones in the case of sections.

References

- [1] L. H. Loomis, H. Whitney, An inequality related to the isoperimetric inequality, Bull. Amer. Math. Soc., 55 (1949), 961–962.
- [2] M. Meyer, A volume inequality concerning sections of convex set, Bull. London Math. Soc., 20 (1988), no. 2, 151–155.

Formal Proof through Type Theory

[Federico Cabrera Linares](#), Autonomous University of Madrid

Type theory lies at the intersection of logic, computation, and the foundations of mathematics. It provides a framework in which propositions correspond to types and proofs to terms, offering both a foundation for constructive mathematics and the basis of modern proof assistants. This talk will introduce the core ideas of type theory following the early chapters of *Type Theory and Formal Proof*, aiming to present its motivations, basic constructions, and its role in contemporary logic and computer science.

References

- [1] Rob Nederpelt and Herman Geuvers (2014) *Type Theory and Formal Proof*, Cambridge University Press, Cambridge.

Proof Theory for Provability Logics: A Uniform Approach

Borja Sierra Miranda, University of Bern

Proof theory is the branch of Mathematical Logic whose main object of study is the concept of proof. Provability logics are formal reasoning systems which allow us to derive general results about the concept of proof formalized inside a mathematical theory, such as Peano Arithmetic or Set Theory.

In the recent years, a new tool has been added to the toolkit of the proof theorist. This is the use of non-wellfounded proofs, i.e., proofs whose chain of reasoning does not need to start at any point and may be infinite. Originally, non-wellfounded proofs were not designed to be used with provability logics. Nevertheless, in [1] and [2] it was proven that they could be used to provide simplified proofs of previously known results and develop new results for the logics **GL** (Gödel-Löb logic), **Grz** (Grzegorczyk logic) and **wGrz** (weak Grzegorczyk logic).

In this talk we will present a generalization and extension of the results of [1] and [2] covering more provability logics. Particularly, we will briefly talk about a simplified proof of cut elimination (a fundamental concept in Proof Theory) that can be done uniformly in multiple provability logics ([3]) via non-wellfounded proofs. If time allows, we will also talk about new results in the direction of uniform/Lyndon interpolation ([4], [5]) that have been proven via the use of non-wellfounded proofs. One of the curiosities of these results is that some of them have been tried to be solved via the use of semantical or wellfounded proof-theoretical methods for a while without any success.

References

- [1] Daniyar Shamkanov (2014) *Circular proofs for the Gödel-Löb provability logic*, Mathematical Notes 96: 575-585.
- [2] Yuri Savateev, and Daniyar Shamkanov (2021) *Non-well-founded proofs for the Grzegorczyk modal logic*, The Review of Symbolic Logic 14(1): 22-50.
- [3] Borja Sierra Miranda, Thomas Studer, and Lukas Zenger (2024) *Coalgebraic proof translations for non-wellfounded proofs*, Advances in Modal Logic 15: 527-548.
- [4] Sebastijan Horvat, Borja Sierra Miranda, and Thomas Studer (2025) *Non-Wellfounded Proof Theory for Interpretability Logic*, Pozzato, G., Uustalu, T. (eds) Automated Reasoning with Analytic Tableaux and Related Methods. TABLEAUX 2025, to be published.
- [5] Iris van der Giessen, Guillermo Menéndez Turata, and Borja Sierra Miranda (202?) *Uniform interpolation for GL and iGL*, to appear.

Proof Mechanization: topological and algebraic methods towards the automatic development of mathematical proofs

Villar Ortega, Javier, Universidad de La Rioja

The work of Per Martin-Löf on the development of functional programming languages that could be used to model the syntax of mathematical proofs has yielded a vast field of study, as it provides a way to interpret mathematical reasoning as a process susceptible to be computerized. That is, were we to be able to translate mathematical proofs into a programming language, through the Curry-Howard Correspondance, we would be able to employ the compilers of such languages as evidence of the veracity of the proofs.

Recent developments, such as the apparition of Homotopy Type Theory as a field of study, have propelled the goals of the program towards both proof assistants, that is, programming languages with the capabilities described above; and the process of certification of programs, that is, through the inverse problem, employing mathematical reasoning to prove the validity of computer code for solving formally specified tasks.

This program is best exemplified through initiatives such as the Univalent Program, that aims to propose a homotopically-inspired new system of foundations for Mathematics, more compatible with proof assistants; the Xena Project, a collaborative initiative to make proof assistants more widely employed; and the ambitious project, led by Kevin Buzzard, to provide a fully verifiable proof of Fermat's Last Theorem. These projects have involved many mathematicians in a collaborative endeavour to mechanize the proofs of vast swaths of Algebraic Number Theory, the computer-assisted development of new proofs for classical results, and the finding of several intractable errors to be corrected in classical proofs.

The development of large language models in past years has also sparked a different trend: that of developing automated agents, able to provide mathematical proofs. While the developments employing general commercially available models, for tasks such as solving Olympiad-level problems, has been rapid, two exciting propositions stem from combining these attempts with a Type Theory driven approach: the development of language models able to produce code in these formalization languages, such as Agda or Lean, so that the compiler could provide a barrier against an automated agent providing hallucinated believable false arguments; and the development of dictionaries of proofs that could be easily searched, for agents to pull technical lemmas from them, facilizing the search for relevant already available theory.

In this short talk, we present a summary of different lines of work towards the achievement of automated agents able to satisfy these goals, and how the theory of the involved programming languages defines and measures the limits and obstacles of these proposals.

References

- [1] The Univalent Foundations Program Homotopy Type Theory. *Arxiv*, [1308.0729], 2013.

- [2] E. Riehl On the ∞ -topos semantics of Homotopy Type Theory. *Arxiv*, [2212.06937], 2022.
- [3] S. Awodey, N. Gambino, K. Sojakova Homotopy-initial algebras in type theory. *Journal of the ACM*, Volume 63, Issue 6, Article 51, p. 1–45, 2017.
- [4] D. Annenkov, P. Capriotti, N. Kraus, C. Sattler Two-level Type Theory and Applications. *Mathematical Structures in Computer Science*, Special Issue: Homotopy Type Theory, 2023, pp. 1-56.
- [5] P. L. Lumsdaine, M. Shulman Semantics of higher inductive types. *Mathematical Proceedings of the Cambridge Philosophical Society*, 169 (2020) 159-208.
- [6] C. Cohen, T. Coquand, S. Huber, A. Mörtberg Cubical Type Theory: a constructive interpretation of the univalence axiom. *Arxiv*, [1611.02108], 2016.
- [7] The Xena Project AI at IMO 2025: a round-up *Xena Project*, , 2025.
- [8] K. Buzzard Fermat’s Last Theorem - how it’s going *Xena Project*, , 2024.
- [9] The Lean Prover Community mathlib *Github*, .
- [10] The Agda Community The Agda Standard Library *Github*, .

Measuring the strength of the theory ECA_0 via the Π_2^1 -norm

Anton Fernández Dejean, TU Vienna

A theory is a collection of formulas given by the logical consequences of its axioms. For example, the well-known PA , Peano Arithmetic, is an axiomatization of natural numbers with sum and multiplication. From simple axioms ruling the behaviors of those operations (e.g., $x + 0 = 0$), we can derive the theorems of PA by applying the usual rules of first-order logic. Our focus will be on ECA_0 , a weak subtheory of second-order arithmetic. Hence, we work with two kind of variables: ordinary variables x for natural numbers and set variables X for sets of natural numbers. Typically, subtheories of second-order arithmetic extend PA by adding axioms stating which sets of natural numbers are assumed to exist.

Historically, questions such as “*How strong is a theory?*” have motivated the research of many logicians. In particular, in Proof Theory, the field of Mathematical Logic that studies formal proofs as mathematical objects, logicians have developed the branch of ordinal analysis, that classifies theories according to some of their consequences (see [2] for an overview). Recently, this has been extended to the study of the Π_2^1 -norm of a theory, that measures the Π_2^1 -consequences (i.e., the theorems of the form $\forall X \exists Y \phi(X, Y)$ where ϕ does not contain any quantifier over set variables) through an object called *dilator*, which extends ordinals in a structured way (see [1]).

In this talk, we will introduce the theory ECA_0 and the proof-theoretic notion of a dilator, and we will show how to compute the Π_2^1 -norm of ECA_0 . The proof avoids hard notions of Proof Theory and mainly relies on structural and combinatorial concepts.

References

- [1] Aguilera, J.P. and Pakhomov, F. (2023), *The Π_2^1 consequences of a theory*. J. London Math. Soc., 107: 1045-1073.
- [2] Rathjen, M.. *The Art of Ordinal Analysis*. In M. Sanz-Solé, J. Soria, J.L. Varona and J. Verdera (eds.), Proceedings of the International Congress of Mathematicians Madrid, August 22–30, 2006, Volume II. European Mathematical Society, 2007.

Idempotent elements of the group algebra

Vicent Miralles Lluch, Universitat Politècnica de València

This talk focuses on the calculation of centrally primitive idempotent elements of the group algebra over finite fields. These elements are fundamental, as they allow the algebra to be decomposed into blocks, each generated by one of these idempotents. Thus, they provide a complete description of its structure [1]. Since many results in representation theory rely on the assumption of an algebraically closed field, we introduce the concept of a splitting field for a group [2], which generalises this notion. We then explore how the Galois action on the group algebra over a splitting field provides a method for obtaining these idempotents from the original field [3].

References

- [1] Charles Curtis and Irving Reiner (1962) *Representation of Finite Groups and Associative Algebras*, New York: Wiley-Interscience.
- [2] Bertram Huppert and Norman Blackburn (1982) *Finite Groups II*, Springer-Verlag.
- [3] Gregory Karpilovsky (1992) *Group Representations Volume 1 Part B Introduction to Group Representations and Characters*, Elsevier Science Publishers B.V.

Automorphisms of Evolution Algebras and their Permutation Representations

Pedro Mayorga Pedraza, Universität Bonn

Evolution algebras are a kind of finite dimensional non-associative algebras first appearing in the field of non-Mendelian genetics. In this talk we are going to discuss some topics relating their automorphisms groups. For the special case of an idempotent evolution algebra, an algebra X with $X^2 = X$, combinatorial tools can be applied. In this way, we will show that said automorphisms group is always finite, following [1]. Then, as original work, following [2], we study some permutation representations, which are morphisms $Aut(X) \rightarrow S_n$. For the most natural permutation representation of the automorphisms group we prove that some restriction appear on the possible images of the representation. We show that highly transitive proper subgroups of S_n can not appear as this image. In addition, we construct another permutation representation, the representation on natural idempotents, which satisfies that any permutation group is realizable through it. This is joint work with Cristina Costoya and Antonio Viruel.

References

- [1] Elduque, A. & Labra, A. Evolution algebras and graphs. J. Algebra Appl. 14 n.7 (2015). <https://doi.org/10.1142/S0219498815501030>
- [2] Costoya, C., Mayorga, P. & Viruel, A. Permutation Representations and Automorphisms of Evolution Algebras. Mediterr. J. Math. 22, 149 (2025). <https://doi.org/10.1007/s00009-025-02924-0>

From Sylow to Hall: Extending Fusion Theory and Exploring Subnormalizers

Luis Pablo Colmenar, Universitat de València

Alexander Moretó, Universitat de València

Noelia Rizo, Universitat de València

In finite group theory, there exists important results which are formulated in terms of Sylow subgroups and are proved using the classical Sylow theorems. Whenever such results appear, it is natural to ask whether their statements can be extended or generalized. One natural direction involves replacing Sylow subgroups with Hall subgroups. In this talk, we will explore how a well-known result by Wielandt serves as a powerful tool for this purpose. We will present two main applications: one related to Alperin's fusion theorem, an essential result in the study of fusion in finite groups, central to tackling local-global problems, and another involving a less familiar concept, the subnormalizer, primarily investigated by Carlo Casolo. The latter connects with new conjectures in character theory and offers new insight into the interaction between group structure and representation theory.

References

- [1] Jonathan L. Alperin (1967) *Sylow intersections and fusion*, Journal of Algebra, **6**(2), 222–241, Academic Press.
- [2] Carlo Casolo (1992) *On the subnormalizer of a p -subgroup*, Journal of Pure and Applied Algebra, **77**(3), 231–238, Elsevier.

G-kernels on Operator Algebras

Marina Polo Rodríguez, KU Leuven

This poster presents key ideas from my Master's thesis on the theory of G -kernels on operator algebras. A G -kernel is defined as a group homomorphism $\alpha : G \rightarrow \text{Out}(A) = \text{Aut}(A)/\text{Inn}(A)$ from a group G into the outer automorphism group of a C^* -algebra A . The gauge symmetries of a system, which are considered redundant from the perspective of observable physical properties, correspond to the inner automorphisms of the algebra describing the system. Consequently, the elements of $\text{Out}(A)$ represent the non-redundant symmetries of the system, making the study of the outer automorphism group crucial for physics.

Every G -kernel is associated with a crucial cohomological invariant, its lifting obstruction, an element in $H^3(G, U(Z(A)))$. A central focus is the realization problem: given a finite group G and a cohomology class in $H^3(G, \mathbb{T})$, can one construct a G -kernel on a UHF-algebra (inductive limits of matrix algebras) with this obstruction?

We explore Izumi's conjecture, which provides a necessary and sufficient condition for such realizations in terms of the order of the obstruction and the supernatural number defining the UHF-algebra. In particular, we outline the proof of the conjecture for finite abelian groups.

Additionally, we discuss how the existing constructions that realize these kernels, such as Connes' construction for cyclic groups, can be understood as AF-actions, which are actions defined via compatible sequences on finite-dimensional algebras.

References

- [1] Kenneth S. Brown. *Cohomology of groups*. Vol. 87. Graduate Texts in Mathematics. Springer-Verlag, New York-Berlin, 1982.
- [2] A. Connes. "Periodic automorphisms of the hyperfinite factor of type II_1 ". In: *Acta Sci. Math. (Szeged)* 39.1-2 (1977), pp. 36-66.
- [3] S. Girón Pacheco. "Anomalous symmetries of classifiable C^* -algebras". PhD thesis. University of Oxford, 2023.
- [4] Masaki Izumi. *G-kernels of Kirchberg algebras*. 2024. arXiv: 2309.03441
- [5] V. F. R. Jones. "An invariant for group actions". In: *Algèbres d'opérateurs (Sém., Les Plans-sur-Bex, 1978)*. Vol 725. Lecture Notes in Math. Springer, Berlin, 1979, pp. 237-253.

Integration methods in Banach spaces

Lucía Castán Anglada, Complutense University of Madrid

This talk provides a historical overview of the evolution of integration theory from its origins to the present. In the 19th century, mathematicians sought a rigorous definition of the integral for scalar-valued functions. Cauchy initiated the idea through approximations with finite sums and Riemann and Darboux formalized these notions, leading to the Riemann integral. However, it showed important limitations, and Weierstrass' attempts to extend it were insufficient. The view of the integral as an “area” led to the development of measure theory. Jordan proposed a measure which lacked of countable additivity and Borel refined the idea with stricter conditions, but restricting to fewer sets. Lebesgue unified these approaches, creating his measure and the Lebesgue integral.

The next challenge was extending integration to Banach space-valued functions. The Riemann integral could be adapted, but with significant losses of classical properties. In the case of Lebesgue, the extension gave rise to the Bochner integral, which is defined as the limit of integrals of approximations by simple functions, preserving key theorems like dominated convergence while failing to satisfy others, such as the Radon–Nikodym theorem.

Interest then shifted to broader spaces beyond Banach, introducing topological vector spaces and quasi-Banach spaces, where major theorems hold in this context, such as the open mapping theorem, while others fail, like Hahn–Banach. The talk will finish studying sufficient and necessary conditions that guarantee the extension of Riemann and Bochner integrals and discussing whether an optimal integration method can be defined in such general spaces.

References

- [1] D. M. BRESSOUD (2008) *A radical approach to Lebesgue's theory of integration*, MAA Textbooks, Cambridge University Press, Cambridge.
- [2] J. DIESTEL AND J. J. UHL, JR. (1977) *Vector measures*, Mathematical Surveys, N^o. 15, American Mathematical Society, Providence, RI. With a foreword by B. J. Pettis.
- [3] R. GORDON (1991), Riemann integration in Banach spaces, Rocky Mountain J. Math., 21, pp. 923– 949.
- [4] N. J. KALTON, N. T. PECK, AND J. W. ROBERTS (1984) *An F -space sampler*, vol. 89 of London Mathematical Society Lecture Note Series, Cambridge University Press, Cambridge.
- [5] W. RUDIN (1991) *Functional analysis*, International Series in Pure and Applied Mathematics, McGraw- Hill, Inc., New York, second ed.

An Introduction to Nonstandard Analysis: Where Infinitesimals Truly Exist

Marc Ventura, Universitat de València

The use of infinitesimal and infinite numbers in mathematics dates back to the origins of calculus. It was not until the 1960s that Robinson [1] provided a rigorous foundation for these ideas through Nonstandard Analysis (NSA). Instead of treating infinitesimals as vague notions, NSA constructs a precise number system extending the real numbers, known as *hyperreal numbers*. While often associated with model theory, there is a more intuitive algebraic approach: the hyperreal numbers can be built explicitly using ultraproducts (see [2]). This talk provides an accessible introduction to NSA for functions of a single real variable, covering constructions, the extension of sets and functions, and the characterization of limits and derivatives through infinitesimals. A central feature of NSA is the *Transfer Principle*, which allows many properties of the real numbers to carry over naturally to the hyperreals. We will also briefly discuss, with examples, how nonstandard methods can be applied in advanced contexts to prove new results or simplify existing proofs.

References

- [1] Abraham Robinson (1966) *Non-standard analysis*, North-Holland Publishing, New York.
- [2] Robert Goldblatt (1998) *Lectures on the Hyperreals. An Introduction to Nonstandard Analysis*, Springer-Verlag, New York.

Dirichlet's Theorem: Primes in arithmetic progressions

Lorenzo María López Domene, University of Alicante

In 1837, Dirichlet proved the existence of infinitely many primes in arithmetic progressions $\{a + nq\}_n$ with $\gcd(a, q) = 1$, thereby confirming a conjecture of Legendre and inaugurating analytic number theory. In this talk, we will introduce the central objects involved in Dirichlet's argument and present a streamlined variation of his original proof, which preserves its essential ideas while making them more transparent.

References

- [Apo76] T. M. Apostol. *Analytic number theory*. Undergraduate Texts in Mathematics. Springer-Verlag, New York-Heidelberg, 1976.
- [Dav00] H. Davenport. *Multiplicative number theory*. Second edition. Graduate Texts in Mathematics, Vol. 74. Springer-Verlag, New York, 2000.
- [Ser73] J. P. Serre. *A course in arithmetic*. Graduate Texts in Mathematics, Vol. 7. Springer-Verlag, New York-Heidelberg, 1973.

Tensor-Triangular Geometry

Javier Herrero Cañedo, University of Bonn

Triangulated categories are a ubiquitous structure in geometry, topology and representation theory. It is often the case that these categories admit a symmetric monoidal structure compatible with the triangulated structure, giving rise to the notion of **tt-categories**. In [2], Balmer defines the spectrum of a tt-category, a construction that resembles the Zariski spectrum of a commutative ring. This framework allows one to apply algebro-geometric methods in diverse contexts such as homotopy theory or modular representation theory.

In this talk, we will introduce the notion of tt-categories and present some fundamental examples: the derived category of a commutative ring and the stable category of a finite group in modular characteristic. After defining the notion of Balmer spectrum and its universal property, we will illustrate the theory by computing this spectrum in the aforementioned examples.

References

- [1] Amnon Neeman (1992), *The Chromatic Tower for $D(R)$* , with an appendix by Marcel Bökstedt. *Topology* 31.3.
- [2] Paul Balmer (2005) *The spectrum of prime ideals in tensor triangulated categories*. *J.Reine Angew. Math.*, 588, pp. 149-168.
- [3] Paul Balmer (2020) *A guide to tensor-triangular classification*, Chapter 4 in *Handbook of Homotopy Theory*,
- [4] Dave Benson, Jon Carlson, and Jeremy Rickard (1997), *Thick subcategories of the stable module category*. *Fundamenta Mathematicae* 153.1, pp. 59-80.
- [5] Robert W. Thomason (1997), *The classification of triangulated subcategories*. *Compositio Mathematica* 105.1, pp. 1-27.

The fixed point index sequence and Shub's conjecture

Alejandro Moreno Becerra, Universidad Complutense de Madrid

The fixed point index of a $\mathcal{C}^1$ map f between manifolds is the topological degree of $\text{id} - f$. This index can be studied for f^n , resulting in a sequence whose growth was shown in [1] to be related with the number of periodic points of f .

Shub's conjecture addresses precisely the relation between the growth rate of this index sequence and that of the number of periodic points. In particular, one asks whether

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \log |\text{ind}(f^n)| \leq \limsup_{n \rightarrow \infty} \frac{1}{n} \log (\#\text{Fix}(f^n))$$

holds. This question remains widely open in dimension two and higher. In this talk, we will analyze certain examples in dimension two where f preserves a foliation, provide a global overview of the most significant results on the subject, and present some new contributions of our own.

References

- [1] M. Shub, D. Sullivan, *A remark on the Lefschetz fixed point formula for differentiable maps*, Topology, Volume 13, Issue 2, 1974, Pages 189-191, ISSN 0040-9383.
- [2] Chow, S.N., Mallet-Paret, J., Yorke, J.A. (1983). *A periodic orbit index which is a bifurcation invariant*. In: Palis, J. (eds) Geometric Dynamics. Lecture Notes in Mathematics, vol 1007. Springer, Berlin, Heidelberg.
- [3] Dold, A. Fixed point indices of iterated maps. Invent Math 74, 419–435 (1983).
- [4] G. Graff, M. Misiurewicz and P. Nowak-Przygodzki, *Periodic points for sphere maps preserving monopole foliations*, Qual. Theory Dyn. Syst. **18** (2019), no. 2.

The Bundle-Theoretic Structure and Computational Modeling of Quantum Contextuality

Sahil Imtiyaz, Basque Center for Applied Mathematics, Bilbao, Spain

Non-locality and contextuality are core non-classical features that enable quantum speedups and advantages. Contextuality underlies entanglement and powers the second quantum revolution. Abramsky and Brandenburger showed that contextuality corresponds exactly to obstructions to global sections, enabling a topological, sheaf-theoretic passage from local to global [2]; further generalizations arise via graphs, hypergraphs, algebraic topology, and probability. Our work [1] gives a geometric generalization of [2] via fiber-bundle/gauge theory. The Hilbert-space-independent framework of evolving simplicial bundles formalizes local–global collaboration. The bundle’s gauge group quantifies non-locality/contextuality—a semidirect product of the fiber’s symmetry group and the base space’s fundamental group—representing the base as a simplicial complex (by covering-space theory) in line with Grothendieck–Galois principles. We associate empirical models with explicit simplicial approximations of the base and introduce a strong collapse (discrete Morse theory) where non-contractible cycles appear as critical simplices and non-trivial cycles as transient virtual loops; their existence quantifies non-locality/contextuality. Topology constrains polyhedral symmetries: local consistency = local symmetry-preserving transformations; global consistency = invariant loop-transformations (possibly via permutations) yielding global symmetry-preserving moves. We apply this to Hardy, Bell, GHZ, PR boxes, Kochen–Specker, KCBS, Svetlichny, Liar cycle, and tetrahedron models, distinguishing logical/possibilistic non-locality (Hardy) from strong contextuality (Kochen–Specker). We also sketch a topologically driven computational model beyond the Turing paradigm, grounded in the topological field theory of data [3] and our prior work [4, 5].

References

- [1] Sahil Imtiyaz, Emanuela Merelli, and Serafim Rodrigues *The Bundle-Theoretic Structure and Computational Modeling of Quantum Contextuality*, accepted in Communications in Computer and Information Science, Springer Nature 2025: See: <https://www.bcamath.org/sites/default/files/inline-files/Contextual>
- [2] Samson Abramsky and Adam Brandenburger *The Sheaf-theoretic Structure of Non-locality and Contextuality*, New Journal of Physics, 13 113036, 2011.
- [3] Mario Rasetti and Emanuela Merelli *The Topological Field Theory of Data: a program towards a novel strategy for data mining through data language*, 2015 J. Phys.: Conf. Ser. 626 012005.
- [4] Sahil Imtiyaz and Emanuela Merelli *Contextual Semantics Machinery*, https://www.bcamath.org/sites/default/files/inline-files/Contextual20Semantics20Machinery_1.pdf

- [5] Sahil Imtiyaz and Serafim Rodrigues *Topology of Empirical Models*, <https://www.bcamath.org/sites/default/files/inline-files/Topology20of20Empirical20Models.pdf>

Eisenstein degeneration of Beilinson–Kato classes

[Javier Polo](#), Universidade de Santiago de Compostela

Óscar Rivero, Universidade de Santiago de Compostela

In this talk, I will present joint work with Oscar Rivero, where we investigate the Euler system of Beilinson–Kato elements in families passing through the critical p -stabilization of an Eisenstein series. Within this framework, we establish an explicit link with the system of circular units, making use of factorization formulas in a setting where several p -adic L -functions vanish.

References

- [1] David Loeffler, Óscar Rivero (2024) *Eisenstein degeneration of Euler systems*, J. Reine Angew. Math. 814.
- [2] Javier Polo, Óscar Rivero (2025) *Eisenstein degeneration of of Beilinson–Kato classes and circular units*, preprint, arXiv:2501.01514

The (classical) Riemann–Hilbert correspondence

[Ángel Molina Navarro](#), Universidad Complutense de Madrid

The (classical) Riemann–Hilbert correspondence establishes a surprising relation between solutions of ordinary differential equations on a complex domain and representations of the fundamental group of that domain.

In this talk, we will review this well-known correspondence and discuss its extension to higher-dimensional complex manifolds, especially in the context of moduli spaces and non-abelian Hodge theory.

References

- [1] Tamás Szamuely, *Galois groups and fundamental groups*, Cambridge University Press, Cambridge, 2009.
- [2] Alexander Thomas, *A gentle introduction to the non-abelian Hodge correspondence*. Enseign. Math. 71 (2025), no. 3/4, pp. 261–334.

Microlocalization and derived categories

Alfonso Márquez Martínez, Universidad de Sevilla

Microlocalization is a useful operation in the theory of D-modules, studied by Sato and fully developed by Kashiwara and Schapira in [1]. Despite its construction being fundamentally analytic, there have been attempts to generalize it to an algebraic context, such as [2]. There, the authors define algebraic microlocalization of a filtered module over a ring with respect to an Ore set of the base ring.

One of my PhD goals is to generalize such construction, developing an algebraic microlocalization functor of sheaves of quasi-coherent modules over a scheme, under certain finiteness assumptions. The aim of the talk is to present the first steps in such path, as well as a short introduction to the topic.

References

- [1] [KS90] Kashiwara, M.; Schapira, P. Sheaves on manifolds. Grundlehren Math. Wiss., 292, Springer-Verlag, Berlin, 1990.
- [2] [AvdBvO89] Asensio, M.J.; Van den Bergh, M.; Van Oystaeyen, F. A new algebraic approach to microlocalization of filtered rings. Trans. Amer. Math. Soc. 316 (1989), no. 2, 537–553.

Rational homotopy theory

Hugo Labella, Universidad Complutense de Madrid

This talk is based on the author's Bachelor's thesis. In it, an overview of the tools necessary to construct rational spaces, as well as the basic concepts of the homotopy theory of commutative differential graded algebras is given. After this, a functor that assigns to every topological space a graded commutative differential algebra is constructed, from this algebra all the information of the rational homotopy type of the space can be extracted. This is done using the original ideas of Sullivan, using polynomial forms in an analogy with the de Rham complex of a differentiable manifold.

We prove this functor induces an equivalence of categories between a (properly constructed) homotopy category of graded commutative differential algebras and the usual homotopy category of rational spaces. Using this correspondence we prove by simple calculations results about the non-torsion components of the homotopy groups of spaces such as spheres or complex projective spaces.

References

- [1] Félix, Yves and Halperin, Stephen and Thomas, J-C (2021) *Rational homotopy theory*, Springer Science & Business Media, vol. 205.
- [2] Griffiths, Phillip and Morgan, John W and Morgan, John W (1981) *Rational homotopy theory and differential forms*, Springer, vol. 16.
- [3] Tralle, Alesky and Oprea, John (2006) *Symplectic manifolds with no Kähler structure*, Springer.

Orbifolds and symmetry groups

Pedro Fidalgo Martínez, Universidad de Valladolid

The concept of orbifold was popularized by Thurston [4] and generalizes the concept of manifold. Instead of being locally Euclidean, orbifolds are locally homeomorphic to the quotient of an open set of $\mathbb{R}^n$ by a finite group of automorphisms. This allows these objects to have well-defined singularities.

Following Conway's ideas [2] we can classify the 17 wallpaper groups using orbifolds. To achieve this we will define the orbifold Euler characteristic χ_{orb} , which will be calculated using the orbifold signature and Conway costs. Conway's so-called *magic* Theorem is the key: it states that orbifolds of the form $\mathbb{R}^2/G$, with G being crystallographic, satisfy $\chi_{\text{orb}}(\mathbb{R}^2/G) = 0$. It turns out that there is a one-to-one correspondence between the wallpaper groups and the orbifold signatures with zero χ_{orb} . For a good summary on this topic see the paper by Conway [1] or [3].

Moreover, we can classify the space point groups G using the same strategy. Now we have $\chi_{\text{orb}}(\mathbb{S}^2/G) > 0$. Further generalizations can be achieved with the frieze groups, the plane hyperbolic groups or the space crystallographic groups.

References

- [1] Conway, J. H. (1992). The orbifold notation for surface groups. In M. W. Liebeck & J. Saxl (Eds.), *Groups, Combinatorics and Geometry* (pp. 438–447). Cambridge University Press. DOI: <https://doi.org/10.1017/CBO9780511629259.038>.
- [2] Conway, J. H., Burgiel, H., & Goodman-Strauss, C. (2008). *The Symmetries of Things* (1st ed.). CRC Press.
- [3] Fidalgo Martínez, Pedro. (2025). *Grupos cristalográficos y orbifolds*. TEMat, 9. 31-44. ISSN: 2530-9633. URL: <https://temat.es/articulo/2025-p31>.
- [4] Thurston, W. P. (2022). *The geometry and topology of three-manifolds*. American Mathematical Society.

Relation between the Riemann and Riemannian surfaces. The Mapping Theorem and the Beltrami equation.

Victor Carrasco Yllanes, University of Sevilla

Francisco Torres de Lizaur, University of Seville

Given a surface, we have several ways of equipping it with a proper structure. For instance, in the real three-dimensional space, we can provide it with charts to the real plane and with the inner product in $\mathbb{R}^3$, giving us a way to measure the lengths of vectors tangent to it. Riemann introduced two types of structured surfaces that bear his name to this day: Riemann and Riemannian Surfaces. But do these surfaces share something more than their confusingly similar name? To answer this question we need to introduce The Mapping Theorem. This result ensures the existence of a quasiconformal solution f (permits the bounded distortion of angles locally) defined in an open set of the complex plane to the Beltrami equation

$$\frac{\partial f}{\partial \bar{z}} = \mu \frac{\partial f}{\partial z} \quad (1)$$

when μ verifies certain regularity conditions.

We start the talk giving a brief introduction to both types of surfaces, adding a bit of historical context. We will then ask ourselves if given any Riemannian surface, we are able to find isothermal coordinates (in which the surface's metric is conformal to the Euclidean) for it. The answer will too lie in the Mapping Theorem. Making use of this result we will show that isothermal coordinates do exist, which will let us provide all orientable Riemannian surfaces with a Riemann surface structure.

We will also briefly touch on Garsia's article [1], which talks about the reciprocal affirmation. This talk is based on my Master's final dissertation, directed by Francisco Torres de Lizaur.

References

- [1] Garsia, Adriano M.. *An Imbedding of Closed RIEMANN Surfaces in EUCLIDEAN Space*. Commentarii mathematici Helvetici 35 (1961): 93-110.
- [2] Donaldson, Simon *Riemann surfaces*. Oxford Graduate Texts in Mathematics, 22. Oxford University Press, Oxford, 2011.
- [3] Hubbard, John Hamal *Teichmüller theory and applications to geometry, topology, and dynamics. Vol. 1. Teichmüller theory*. Matrix Editions, Ithaca, NY, 2006.

A method to associate non-associative algebras with combinatorial structures

Jesús Baena Gómez, Dpto. de Geometría y Topología. Facultad de Matemáticas. Universidad de Sevilla. Calle Tarfia, s/n, 41012-Seville (Spain).

Manuel Ceballos, Dpto. de Ingeniería. Universidad Loyola Andalucía. Av. de las Universidades, s/n, 41704 Dos Hermanas, Sevilla (Spain).

Desamparados Fernández-Ternero, Dpto. de Geometría y Topología. Facultad de Matemáticas. Universidad de Sevilla. Calle Tarfia, s/n, 41012-Seville (Spain).

In this paper, we propose a method to associate finite-dimensional non-associative algebras, defined over a fixed basis, with combinatorial structures which are directed graphs that potentially contain full triangles. Moreover, three algorithmic tools are introduced. The first one checks whether a given algebra belongs to the considered class. The second one, which is a modification of the first one, determines if a combinatorial structure is associated with an algebra of this class and, if so, provides the additional constraints that the edge weights must satisfy. The last one tests whether any two algebras are isomorphic and returns the corresponding change of basis if so.

In particular, we apply these algorithmic procedures to the class of Tortkara algebras. Tortkara algebras are a novel class of non-associative algebras introduced by A. S. Dzhumadil'daev in [1], who showed that every Zinbiel algebra with the commutator as multiplication is a Tortkara algebra. These algebras satisfy an identity that combines symmetry and alternation. Research has since focused on their structural properties, relationships with Zinbiel and other non-associative algebras, and their classification under various conditions (see [2] and [3]).

References

- [1] A. Dzhumadil'daev. Zinbiel algebras under q -commutators. *J. Math. Sci.*, 144(2):3909–3925, 2007.
- [2] I. Gorshkov, I. Kaygorodov, and M. Khrypchenko. The geometric classification of nilpotent Tortkara algebras. *Comm. Algebra*, 48(1):204–209, 2020.
- [3] I. Gorshkov, I. Kaygorodov, and M. Khrypchenko. The algebraic classification of nilpotent Tortkara algebras. *Comm. Algebra*, 48(8):3608–3623, 2020.

Novikov algebras and combinatorial structures

Carmen Gutiérrez Silva, Dpto. de Ingeniería Universidad Loyola Andalucía

Manuel Ceballos González, Dpto. de Ingeniería Universidad Loyola Andalucía

In this paper, we study the relationship between low-dimensional Novikov algebras and combinatorial structures. Our focus is on the association of pseudodigraphs that encode their structure constants. We determine which configurations correspond to these algebras through a detailed analysis of admissible and forbidden cases in dimensions 1, 2, and 3. From the structure of the associated graphs, we also deduce several algebraic properties.

Moreover, the isomorphism classes of each 2-dimensional configuration associated with these algebras is analyzed. In order to complement the theoretical study, an algorithm implemented in Maple is presented, capable of automatically generating the law of a Novikov algebra from a given combinatorial structure and verifying the satisfaction of the Novikov identities. Finally, the computational complexity of this algorithm is studied.

Keywords: Digraph, Pseudodigraph, Combinatorial structure, Novikov algebras.

2020 Mathematics Subject Classification: 17D99 05C25, 05C20, 05C22.

References

- [1] C.M. Bai and D.J. Meng (2001) *L^AT_EX: The classification of Novikov algebras in low dimensions*, Journal of Physics A: Mathematical and General.
- [2] A. Carriazo, L.M. Fernández and J. Núñez (2004) *L^AT_EX: Combinatorial structures associated with Lie algebras of finite dimension*, Linear Algebra and its Applications.
- [3] M. Ceballos and J. Núñez and A. F. Tenorio (2018) *L^AT_EX: Finite-dimensional Leibniz algebras and combinatorial structures*, Communications in Contemporary Mathematics.
- [4] I. Karimjanov, I. Kaygorodov and A. Khudoyberdiyev (2019) *L^AT_EX: The algebraic and geometric classification of nilpotent Novikov algebras*, Journal of Geometry and Physics.
- [5] P. Dechant, Y. He, E. Heyes and E. Hirst (2023) *L^AT_EX: Cluster algebras: Network Science and Machine Learning*, Journal of Computational Algebra.

The Structure of Symmetric Polynomials

José Molina, Universidad de Sevilla

In this talk, we will discuss symmetric polynomials, working with the polynomial algebra over an arbitrary field and any number of variables. In particular, we will introduce the elementary symmetric functions and the lexicographic order on monomials. These concepts will allow us to prove our main goal: the Fundamental Theorem of Symmetric Polynomials.

References

- [1] David Cox, John Little, and Donal OShea (2013) *Ideals, varieties, and algorithms: an introduction to computational algebraic geometry and commutative algebra*, Springer Science and Business Media.
- [2] José Molina (2023) *An approach to the study of invariant polynomials under group actions*, Bachelor's thesis, Universidad de Málaga.

The Mordell-Weil Theorem

Carlos García Ordóñez, Universidad Complutense de Madrid

The Mordell-Weil Theorem is one of the fundamental results of arithmetic geometry, and a starting point for much of the modern theory in elliptic curves. This theorem asserts that the rational points of an elliptic curve defined over a number field are always a finitely generated group. The main objective of this talk is to present the main ideas of its proof, namely, the weak Mordell-Weil theorem, the descent procedure and the theory of heights in elliptic curves.

In order to do so, the talk will first introduce the basic theory of elliptic curves and analyse its algebraic and arithmetic properties. Then some ideas from number theory and elliptic curves over p -adic fields will be given, finishing with the reduction theory of elliptic curves. All these tools will be put together to prove the Mordell-Weil theorem, and an explicit computation of the rank of an elliptic curve will be done to illustrate the methods known.

Time-permitting, a cohomological reinterpretation of the Mordell-Weil theorem in terms of localizations and Selmer group will be given; as well as very brief introduction to modular forms and the BSD conjecture.

References

- [1] J. Silverman (2009) *The arithmetic of elliptic curves*, Springer, 2nd Edition
- [2] J. Silverman (1994) *Advanced topics in the arithmetic of elliptic curves*, Springer
- [3] J. Neukirch (2013) *Algebraic Number Theory*, Springer
- [4] F. Diamond, J. Shurman (2006) *A First Course in Modular Forms*, Springer

Wedderburn-Artin Theory

Jorge Casanova Moya, Master student at Universidad Complutense de Madrid

A cornerstone of non-commutative algebra is the Artin–Wedderburn theorem, which shows that every semisimple ring is a finite product of matrix rings over division rings. Its proof relies heavily on module theory, a branch of math heavily used by algebraic geometers and illustrates the deep connection between the structure of rings and their representations.

The study of finite-dimensional associative algebras further enriches this perspective. The elegant classification of finite dimensional real division algebras and finite division rings demonstrates how these objects extend familiar number systems.

Tensor products offer a unifying framework, leading to fundamental results such as the Skolem–Noether theorem and the double centralizer theorem. Together, these ideas serve as an invitation to non-commutative algebra, highlighting its elegance, coherence, and far-reaching significance.

References

- [1] Benson Farb and R. Keith Dennis. *Noncommutative Algebra*. Springer, Graduate Texts in Mathematics, 2020. ISBN: 978-1-0716-0893-2. DOI: 10.1007/978-1-0716-0894-9.
- [2] Matej Brešar. *Introduction to Noncommutative Algebra*. Springer, Universitext, 2014. ISBN: 978-3-319-08692-7. DOI: 10.1007/978-3-319-08693-4.
- [3] Matej Brešar. *The Wedderburn-Artin Theorem* [Preprint]. arXiv:2405.04588, 2024. Disponible en: <https://arxiv.org/abs/2405.04588>
- [4] Sharifi, Y. (s.f.), *Noncommutative Ring Theory*. YSharifi’s blog. Retrieved from <https://ysharpifi.wordpress.com/category/noncommutative-ring-theory/>

Operator algebras and modular entropy in holography.

Franco Salinas Pytel, Universidad Complutense de Madrid

Entanglement is one of the defining properties of finite-dimensional quantum systems, often quantified by the von Neumann entropy. In this talk, I will discuss how this notion extends to quantum systems with infinitely many degrees of freedom, as described by quantum field theories (QFT), through the concept of modular entropy. The motivation comes from holographic duality, specifically the AdS/CFT correspondence, which relates the geometry of anti-de Sitter spaces to conformal field theories. This perspective naturally leads to the introduction of Tomita–Takesaki modular theory within the framework of von Neumann algebras. I will conclude by commenting on applications of modular entropy, with particular emphasis on its relation to the computation of Rényi entropies in holography.

References

- [1] Ola Bratteli and Derek W. Robinson, *Operator Algebras and Quantum Statistical Mechanics 1: C^* - and W^* -Algebras. Symmetry Groups. Decomposition of States*, 2nd ed., Springer, Berlin / Heidelberg, 1987.
- [2] Edward Witten, “Notes on Some Entanglement Properties of Quantum Field Theory,” *Rev. Mod. Phys.*, vol. 90, no. 4, Article 045003, 2018; arXiv:1803.04993.
- [3] Daniel Harlow, “TASI Lectures on the Emergence of the Bulk in AdS/CFT,” arXiv:1802.01040 [hep-th], 2018.
- [4] Horacio Casini, Marina Huerta, and Robert C. Myers, “Towards a derivation of holographic entanglement entropy,” *JHEP*, vol. 2011, no. 5, Article 036, 2011; arXiv:1102.0440 [hep-th].

Spectral properties of Hausdorff operators on Fock spaces

Jaime Arto Alseda, University of Zaragoza

Fock spaces (denoted by F_α^p) are Banach spaces of entire functions. In 2024, O. Blasco published the article *Boundedness and compactness of Hausdorff operators on Fock spaces*, where he studies conditions that characterize the boundedness and compactness of the Hausdorff operator $\mathcal{H}_\mu$ over F_α^p , where μ is some positive Borel measure over $(0, \infty)$. The importance of this operator arises from the fact that it is the natural generalization of the classic Cesàro operator.

Under those conditions, $\mathcal{H}_\mu$ can be written as the integral of an operator semigroup $(T(t))_{t \geq 0}$. Moreover, if $p < \infty$, then that semigroup happens to be strongly continuous.

The theory of operator semigroups provides us with powerful tools that will allow us to compute the fine spectra of the Hausdorff operator. To do so, we first study the spectra of the infinitesimal generator A of the semigroup $(T(t))_{t \geq 0}$, and then we transfer these spectral properties to the operator $\mathcal{H}_\mu$ via the Hille-Phillips functional calculus and the spectral mapping theorems. As a consequence, we obtain the spectral sets of the Cesàro operator over F_α^p .

References

- [1] L. ABADÍAS, J. OLIVA-MAZA, *Spectral sets of generalized Hausdorff matrices on spaces of holomorphic functions on D* . Journal of Functional Analysis **286** (2024) 110298,
- [2] O. BLASCO, *Boundedness and compactness of Hausdorff operators on Fock Spaces*. Transactions of the American Mathematical Society **377** (2024), 5165-5196,
- [3] K. ENGEL, R. NAGEL, *One-Parameter semigroups for linear evolution equations*. Graduate Texts in Mathematics, **194**, Springer-Verlag (2000), ISBN 978-0-387-22642-2,
- [4] M. HAASE, *The functional calculus for sectorial operators*. Operator Theory: Advances and Applications, **169**, Birkhäuser-Verlag (2006), ISBN 978-3-7643-7697-0,
- [5] K. ZHU, *Analysis on Fock spaces*. Graduate Texts in Mathematics, **263**, Springer (2010), ISBN 978-1-4419-8801-0.

Tournaments and finite spaces

Inés Mora Caro, Dpto. de Geometría y Topología. Facultad de Matemáticas. Universidad de Sevilla. Calle Tarfia, s/n, 41012-Seville (Spain).

Desamparados Fernández Ternero, Dpto. de Geometría y Topología. Facultad de Matemáticas. Universidad de Sevilla. Calle Tarfia, s/n, 41012-Seville (Spain).

A large number of researchers have studied the problem of topologization of combinatorial structures, for example in [3] and [4]. Within the framework of topologies defined on locally finite graphs, the graphic topology was developed in [2] and [1].

In this work, we study the graphic topology defined on finite indecomposable tournaments (complete digraphs), begun in [1]. We deduce a characterization of indecomposable tournaments with few vertices. Finally, we verify that the minimum number of vertices such that there exist non-isomorphic indecomposable tournaments with homeomorphic graphic topologies is six.

References

- [1] J. Dammak and R. Salem, Graphic topology on tournaments, *Adv. Pure Appl. Math.* **9**(4): 279–285, 2018.
- [2] S. M. Jafarian Amiri, A. Jafarzadeh and H. Khatibzadehan, Alexandroff topology on graphs, *Bull. Iranian Math. Soc.* **39**(4): 647–662, 2013.
- [3] D. Nogly and M. Schladt, Digital Topology on Graphs, *Comput. Vis. Image Und.*, **63**(2): 394–396, 1996.
- [4] H. O. Zomam, Out-graphic topology on directed graphs, *J. Math. Comput. Sci.* **13** Article ID 14, 2023.

Additive partial matchings and resolutions of persistent homology modules

Marco Delgado, Departamento de Matemática Aplicada I, Universidad de Sevilla

A persistence module $M : \mathcal{C} \rightarrow_k$ is a functor from a small category $\mathcal{C}$ (usually a poset with its usual categorical structure, or $\mathbb{R}$) to the category of vector spaces over a fixed field k . The category of persistence modules $\mathcal{C}_k$ is an abelian category, since it is a functor category from a small category to an abelian one. We consider persistence $\mathbb{R}$ -modules, i.e., the case where $\mathcal{C} = \mathbb{R}$ with the usual categorical structure of a totally ordered set. When a persistence module is pointwise finite-dimensional (p.f.d.) i.e $M(t)$ is finite-dimensional for every $t \in \mathbb{R}$, it can be decomposed uniquely (up to isomorphism) as a direct sum of indecomposable interval modules with local endomorphism ring. Such modules are completely described by a multiset called their *barcode* [3]. Given two persistence modules U and V , a *persistence morphism* $f : V \rightarrow U$ is a natural transformation. A natural question is whether a morphism $f : V \rightarrow U$ between p.f.d. modules induces a relation, specifically, a partial matching, between their barcodes. It is known that such partial matchings cannot be functorial [2]. Recently, a *block function* $\mathcal{M}_f$ associated with the morphism f was introduced [1]. This function is algebraically well-defined, linear with respect to direct sums of persistence morphisms, efficiently computable via matrix-reduction techniques, and induces a partial matching between the barcodes of U and V . However, for arbitrary p -dimensional homology, additive partial matchings are harder to understand. In this paper, we take a first step toward a stability proof for the arbitrary-dimensional case of additive partial matchings. More precisely, we establish that the p -dimensional persistent homology module of the Vietoris-Rips filtration of a pointcloud is linked to the 0-dimensional persistent homology of a solution to the *inverse barcode problem*. This allows us to recover the former from the latter using algebraic techniques and a specific persistence-module resolution. We further show that certain resolutions of 0-dimensional persistent homology preserve additive partial matchings, enabling us to bound p -dimensional matchings in terms of well-behaved 0-dimensional ones.

References

- [1] R. Gonzalez-Diaz, M. Soriano-Trigueros, A. Torras-Casas (2024) *Additive Partial Matchings Induced by Persistence Morphisms*, arXiv: 2006.11100.
- [2] U. Bauer, M. Lesnick (2013) *Induced Matchings of Barcodes and the Algebraic Stability of Persistence*, Proc. Annu. Symp. Comput. Geom., doi: 10.1145/2582112.2582168.
- [3] M.B. Botnan, W. Crawley-Boevey (2018) *Decomposition of persistence modules*, Proc. Am. Math. Soc., doi: 10.1090/proc/14790.

Non-intersecting paths and the determinant of the distance matrix of a tree

L. Esquivias-Quintero, University of Seville

E. Briand, University of Seville

A. Gutiérrez, University of Bristol

J. Lillo, University of Seville

M. Rossas, University of Seville

Consider a tree T with vertices labeled from 1 to n , and edge set E . The distance between vertices i and j , denoted by $d(i, j)$, is defined as the number of edges along the unique path connecting them in T . The distance matrix of T , denoted by $M(T)$, is then defined as $(d(i, j))_{1 \leq i, j \leq n}$. In an influential 1971 paper, Graham and Pollak established that the determinant of the distance matrix of T obeys what is now known as the Graham–Pollak formula:

$$\det M(T) = (-1)^{n-1} (n-1) 2^{n-2}.$$

Observe that this formula implies that the determinant of the distance matrix of T is solely dependent on its number of vertices, and not on its tree structure.

Multiple techniques from linear algebra, ranging from Gaussian elimination to Charles Dodgson’s condensation formula, have been used to prove the Graham–Pollak formula.

The expression $(n-1)2^{n-2}$ strongly suggests that $\det M(T)$ enumerates something. In 2010, Tillia observed that since a tree with n vertices has $n-1$ edges, the expression $(n-1)2^{n-2}$ counts the number of ways to select one edge of T and assign orientations to the remaining $n-2$ edges. This work establishes a combinatorial framework that validates this observation. An approach not only unifies existing generalizations and q -analogues of the formula but also paves the way for deriving new results.

References

- [1] Emmanuel Briand, Luis Esquivias-Quintero, Álvaro Gutiérrez, Adrián Lillo, Mercedes Rosas (2024) *Non-intersecting paths and the determinant of the distance matrix of a tree*, Arxiv, <https://doi.org/10.48550/arXiv.2407.01227>
- [2] Ira Gessel and Gérard Viennot (1985) *Binomial determinants, paths, and hook length formulae*. Adv. in Math., 58(3):300–321. doi:10.1016/0001-8708(85)90121-5.
- [3] Ronald L. Graham and Henry O. Pollak (1971) *On the addressing problem for loop switching*. Bell System Tech. J., 50:2495–2519. doi:10.1002/j.1538-7305.1971.tb02618.x.

Posters

Polynomial bases and homomorfisms on representations of the symmetric group

Aarón Ocampo Amaya, Universidad de Sevilla

This talk/poster is meant to show some original results based on well known contructions in field of representation theory. We intend for this presentation to be understandable from an entry level point of view. We'll take the time to introduce the theory and motivate the results, while avoiding the more tecnical details. A summary of the presentation is now given.

Polynomial bases for representations of the symmetric group $\mathbb{S}_n$ have been studied extensively since the introduction of higher Specht polynomials [1]. These polynomials serve as a nice way of both computationally working with representations, as well as, showing the fundamental combinatorial aspects underlying the theory of $\mathbb{S}_n$ representations.

The regular Specht polynomials can be constructed as a polynomial base for the irreducible component V_λ in the space of polytabloids M_λ for each partition $\lambda \vdash n$ as in [2]. However, as these polytabloid spaces M_λ contain other irreducible components V_μ for $\mu \trianglelefteq \lambda$; they must also contain polynomial bases for these other irreducible components. While similar, these are different from the higher Specht polynomials.

This work explicitly computes these other new polynomials, which turn out to be products of the regular Specht polynomials and certain Schur polynomials. Multilplying by these Schur polynomials, now gives homomorfisms of $\mathbb{S}_n$ -representations $V_\mu \longrightarrow M_\lambda$.

We then use these results to compute homomorfisms between irreducible $\mathbb{S}_n$ - and $\mathbb{S}_{n+1}$ -representations, on some simple cases. These kinds of homomorfisms appear in the theory of FI-modules introduced by CEF [3]. While the main theory of FI-modules focuses on studying stability on some representation sequences, we hope to expand this idea onto a more general setting; that gives another interpretation of the so called "hook stability" portrayed in [4].

References

- [1] Tomohide Terasoma and Hirofumi Yamada. Higher specht polynomials for the symmetric group. *Proc. Japan Acad. Ser. A Math. Sci.*, 69(10):41–44, 1993.
- [2] Bruce Sagan. *The symmetric group: representations, combinatorial algorithms, and symmetric functions*, volume 203. Springer Science & Business Media, 2001.
- [3] T. Church, Jordan S. Ellenberg, and B. Farb. FI-modules and stability for representations of symmetric groups. *Duke Mathematical Journal*, 164(9):1833 – 1910, 2015.
- [4] Emmanuel Briand, Amarpreet Rattan, and Mercedes Rosas. On the growth of the krocker coefficients, ArXiv e-prints, July 2016, 1607.02887.

Graph approximating metric spaces and a new application to dynamical systems

Amaia Zudaire Alducin, Universidad Complutense de Madrid

This work explores the construction of approximating graphs for metric spaces, following the method introduced by Kanai. We study how such graphs, which are quasi-isometric to the original space, preserve key geometric properties, including p -parabolicity, geodesic ball growth, the linear isoperimetric inequality and the doubling condition. These invariants allow us to translate problems from continuous to discrete settings. In the final section, we outline a future research direction with some preliminary results: the study of discrete dynamical systems on metric spaces via their induced dynamics on approximating graphs.

References

- [1] Álvaro Martínez-Pérez and José M. Rodríguez (2021) *On p -parabolicity of Riemannian manifolds and graphs*, Revista Matemática Complutense, vol. 35, no. 1, pp. 179–198, Springer. doi:10.1007/s13163-021-00387-x.
- [2] Masahiko Kanai (1985) *Rough isometries, and combinatorial approximations of geometries of non compact Riemannian manifolds*, Journal of the Mathematical Society of Japan, vol. 37, no. 3, pp. 391–413. doi:10.2969/jmsj/03730391.
- [3] Masahiko Kanai (1986) *Rough isometries and the parabolicity of Riemannian manifolds*, Journal of the Mathematical Society of Japan, vol. 38, no. 2, pp. 227–238. doi:10.2969/jmsj/03820227.
- [4] Álvaro Martínez-Pérez and José M. Rodríguez (2018) *Cheeger isoperimetric constant of Gromov hyperbolic manifolds and graphs*, Communications in Contemporary Mathematics, vol. 20, no. 5, 1750050, World Scientific. doi:10.1142/s021919971750050x.
- [5] Álvaro Martínez-Pérez and José M. Rodríguez (2024) *Parabolicity and Cheeger’s constant on graphs*, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas, vol. 118, no. 4, Springer. doi:10.1007/s13398-024-01639-3.

Can Polynomials Produce the Primes?

Prime-Enumerating Polynomials

[Ana Carrasco Martín](#), Universidad de Sevilla

Manuel Jesús Soto Prieto, Departamento de Matemática Aplicada I, Universidad de Sevilla

Primes are known to be the building blocks of numbers, and their beauty has fascinated mathematicians since the very beginning of time. While numerous formulas defining primes are found in the literature, can they be elegantly expressed by a polynomial?

In this poster, based on my undergraduate dissertation ([1]), we will briefly show why no non-constant polynomial can produce only primes. And still—against all intuition—there exist multivariable polynomials whose positive values are exactly the set of prime numbers.

These remarkable constructions, which we call prime enumerating polynomials, connect algebraic number theory, logic, and computability in a striking way, and are linked to the negative solution of Hilbert’s Tenth Problem.

The poster aims to present these surprising ideas in a clear, visual, and engaging manner, combining rigorous mathematics with an accessible narrative to invite the audience into the unexpected beauty of this phenomenon.

References

- [1] Carrasco Martín, A. (2024). Polinomios enumeradores de primos. (Trabajo Fin de Grado Inédito). Universidad de Sevilla, Sevilla.

Kuratowski's Theorem

Arianna Charrère, Alma Mater Studiorum Università di Bologna, Master student in advanced mathematics for applications

Kuratowski's Theorem is a cornerstone result in the characterization of planar graphs. The Theorem states that a graph is planar if and only if it does not contain as a subgraph any subdivision of K_5 or $K_{3,3}$. Thanks to Euler's formula it is easy to demonstrate the sufficient condition of planarity. The necessary condition is more technical and requires more additional propositions and lemmas that give a better understanding of the planar graph's structure. In this poster, we are going to give a sketch of its proof, given for the first time by Kuratowski himself, which is usually left out of books and lectures.

References

- [1] Yifan Xu, *Kuratowski's Theorem*. <https://math.uchicago.edu/~may/REU2017/REUPapers/Xu,Yifan.pdf>
- [2] S. Tamar-Mattis, *Planar graphs and Wagner's and Kuratowski's Theorem*. <https://math.uchicago.edu/~may/REU2016/REUPapers/Tamar-Mattis.pdf>
- [3] Robin J. Wilson, *Introduction to Graph Theory. Forth edition*. Longman Group Ltd, 1996

The Galton-Watson Process: Theory and Applications

Carlos Cañada Moreno, Universidad de Extremadura

This talk provides a comprehensive mathematical treatment of the Galton-Watson branching process, a fundamental stochastic model for population dynamics. The process is defined recursively where each individual reproduces independently according to a common offspring distribution $\{p_k\}$, forming a homogeneous Markov chain with transition probabilities given by convolution powers of the reproduction law.

The extinction probability q is characterized as the smallest non-negative root of the equation $f(s) = s$, where f is the probability generating function. The process exhibits distinct asymptotic behaviors across three regimes: subcritical ($m < 1$), critical ($m = 1$), and supercritical ($m > 1$). Critical processes satisfy $\lim_{n \rightarrow \infty} P[z_n/n \leq u \mid z_n > 0] = 1 - \exp(-2u/\sigma^2)$ when $\sigma^2 < \infty$. Supercritical processes exhibit martingale convergence $w_n = z_n/m^n \rightarrow w$ almost surely, with L^1 -convergence if and only if $\sum p_k k \log k < \infty$.

The analysis employs generating functions, martingale theory, and limit theorems to derive precise asymptotic results. Additional topics include hitting time distributions and total progeny analysis, rate of convergence theorems.

References

- [1] Jagers, P. *Branching Processes and Their Applications*. Cambridge University Press, 1985.
- [2] Feller, W. *An Introduction to Probability Theory and Its Applications, Volume II*. Wiley, New York, 1966.
- [3] Harris, T. E. *The Theory of Branching Processes*. Springer, Berlin, 1963.
- [4] Athreya, K. B. and Ney, P. E. *Branching Processes*. Springer-Verlag, Berlin, 1972.
- [5] Kesten, H., Ney, P., and Spitzer, F. *The Galton-Watson process with mean one and finite variance*. Teor. Veroyatnost. i Primenen, 11:579-611, 1966.

Dimensionality Reduction in Diffeomorphic Registration under the Euler–Poincaré Equation

Carlos Paesa-Lia, Department of Computer Science and Systems Engineering, Aragon Institute on Engineering Research (I3A), Universidad de Zaragoza

Mónica Hernández, Department of Computer Science and Systems Engineering, Aragon Institute on Engineering Research (I3A), Universidad de Zaragoza

In recent years, diffeomorphic registration methods have become the reference standard within the field of non-rigid medical image registration. In 2005, the Large Deformation Diffeomorphic Metric Mapping (LDDMM) method established the foundations for computing diffeomorphic transformations between images. In 2012, a geodesic shooting algorithm was proposed that constrains the LDDMM problem with the Euler–Poincaré equation (EPDiff), allowing the problem to be reduced to the space of initial velocities and enabling the computation of true geodesics between images. Nevertheless, the high computational cost represents the main limitation for the adoption of these algorithms in large-scale computational anatomy studies. In this work, several model order reduction methods are investigated to address this problem: the band-limited (BL) vector field parametrization of Zhang et al. [1], and the proper orthogonal decomposition (POD) of Wang et al. [2]. Using a modularized software library, comparative experiments are conducted on two different databases extensively used in non-rigid registration evaluation, in order to compare their performance and draw conclusions regarding the effectiveness of the proposed reductions.

References

- [1] Miaomiao Zhang and P. Thomas Fletcher (2015) *Finite-dimensional Lie algebras for fast diffeomorphic image registration*, In *Information Processing in Medical Imaging: 24th International Conference, IPMI 2015, Sabhal Mor Ostaig, Isle of Skye, UK, June 28–July 3, 2015, Proceedings 24*, Springer, pp. 249–260.
- [2] Jian Wang, Wei Xing, Robert M. Kirby, and Miaomiao Zhang (2019) *Data-driven model order reduction for diffeomorphic image registration*, In *Information Processing in Medical Imaging: 26th International Conference, IPMI 2019, Hong Kong, China, June 2–7, 2019, Proceedings 26*, Springer, pp. 694–705.
- [3] Rubén Muñoz Sierra, Mónica Hernández Giménez, and Elvira Mayordomo Cámara (2022) *Descomposición Ortogonal Propia en registro mediante difeomorfismos*, Universidad de Zaragoza.

Mathematical modeling and numerical simulation of batteries

Eva Garijo Alcalde, Instituto de Matemática Interdisciplinar, UCM (evgarijo@ucm.es)

Juan Antonio Infante del Río, Instituto de Matemática Interdisciplinar, UCM

Benjamin Ivorra, Instituto de Matemática Interdisciplinar, UCM

Ángel Manuel Ramos del Olmo, Instituto de Matemática Interdisciplinar, UCM

Over the past few years, lithium-ion batteries have become a key technology for energy storage, proving highly useful in diverse applications ranging from portable electronic devices to electric vehicles and large-scale storage systems. Accurate modeling of these batteries is crucial to improving their performance, extending their lifetime, and ensuring their safety. Various approaches exist for mathematically modeling lithium-ion batteries, including electrochemical, equivalent circuit, and data-driven models. Each method has particular characteristics and is applied in specific contexts.

This short talk presents the studies and results derived from work employing electrochemical models, which offer a detailed description of the physical and chemical processes occurring inside a battery [1]. These models consider phenomena such as ion transport, intercalation/deintercalation reactions, and the formation of SEI (solid electrolyte interphase) layers on the electrodes. One of the main advantages of electrochemical models is their ability to offer detailed insights into the internal behavior of the battery, making them ideal for the analysis and optimization of cell and material design.

Some of their main features include high accuracy in representing internal phenomena, balanced by the need for detailed knowledge of material parameters and properties. The simulations presented here have been obtained using the solvers *DandeLiion* [2], a fast solver for the Newman model, and *COMSOL Multiphysics* [3] for comprehensive multiphysics modeling.

References

- [1] G-W. Richardson, J.M. Foster, R. Ranom, C.P. Please & A. M. Ramos, *Charge transport modelling of Lithium-ion batteries*, European Journal of Applied Mathematics , Volume 33 , Issue 6 , December 2022 , pp. 983 - 1031.
- [2] I. Korotkin, S. Sahu, S. E. J. O’Kane, G. Richardson, and J. M. Foster, "DandeLiion v1: An Extremely Fast Solver for the Newman Model of Lithium-Ion Battery (Dis)charge," *Journal of The Electrochemical Society*, vol. 168, no. 6, p. 060544, Jun. 2021.
- [3] COMSOL, *COMSOL Multiphysics*, Version 6.3, COMSOL Inc., 2024.

Convolutional Neural Networks for Privacy Preservation through Homomorphic Encryption

Francisco Manuel López López, Universidad Politécnica de Madrid

Preserving privacy in data processing is an increasingly relevant challenge in fields such as healthcare, banking, and artificial intelligence, where the use of sensitive information can compromise the confidentiality of individuals. Homomorphic encryption emerges as a promising solution, since it allows operations to be performed on encrypted data without needing to decrypt it first. In this way, it becomes possible to carry out complex computations without ever exposing the original information to the entity performing the calculation. This approach contrasts with traditional systems, where data must be decrypted before processing and therefore becomes vulnerable to potential security breaches.

In particular, lattice-based homomorphic schemes such as CKKS enable secure work with floating-point numbers. These systems rely on mathematically hard problems whose computational intractability ensures cryptographic robustness. At the same time, convolutional neural networks have demonstrated exceptional performance on image classification tasks and structured data processing. However, adapting the conventional architecture of such networks to operate on encrypted data requires transforming both linear and nonlinear operations to comply with the constraints imposed by homomorphic encryption.

This work focuses on designing and implementing a convolutional neural network capable of performing inference directly on encrypted data. To achieve this goal, we first study the mathematical foundations of lattices and homomorphic encryption schemes, as well as the principles of convolutional networks. Next, we examine the intersection of these two technologies by analyzing how to adapt the fundamental operations of these networks to integrate them within the homomorphic encryption framework. A concrete case study is then defined to illustrate a scenario in which privacy is paramount, and the most appropriate library for this purpose is selected. Finally, we evaluate comparative results between inference on plaintext data and homomorphic inference, discussing the implications in terms of accuracy and computational cost, and conclude with the advantages and limitations of integrating convolutional networks with homomorphic encryption as a privacy-preserving solution in real-world applications

References

- [1] Chris Peikert (2015). *EECS 598: Lattices in Cryptography*, lecture notes, Department of Electrical Engineering and Computer Science, University of Michigan. Available at: <https://web.eecs.umich.edu/~cpeikert/lic15/>
- [2] Oded Regev (2010). *The Learning with Errors Problem (LWE)*, Courant Institute of Mathematical Sciences, New York University, invited survey. Available at: <https://cims.nyu.edu/~regev/papers/lwesurvey.pdf>
- [3] Vadim Lyubashevsky, Chris Peikert and Oded Regev (2013). *On Ideal Lattices and Learning with Errors over Rings*, *Journal of the ACM*, 60(6), Article 43, 43:1–43:35. doi: 10.1145/2535925
- [4] J. H. Cheon, A. Kim, M. Kim and Y. Song (2016). *Homomorphic Encryption for Arithmetic of Approximate Numbers*, Cryptology ePrint Archive, Report 2016/421. Available at: <https://eprint.iacr.org/2016/421>
- [5] Michael A. Nielsen (2015). *Neural Networks and Deep Learning*, Determination Press (online book). Available at: <http://neuralnetworksanddeeplearning.com>
- [6] Ran Gilad Bachrach, Nathan Dowlin, Kim Laine, Kristin Lauter, Michael Naehrig and John Wernsing (2016). *CryptoNets: Applying Neural Networks to Encrypted Data with High Throughput and Accuracy*, in *Proceedings of the 33rd International Conference on Machine Learning (ICML 2016)*, PMLR 48, pp. 201–210. Available at: <https://proceedings.mlr.press/v48/gilad-bachrach16.html>
- [7] R. Podschwadt, D. Takabi, P. Hu, M. H. Rafiei and Z. Cai (2022). *A Survey of Deep Learning Architectures for Privacy Preserving Machine Learning With Fully Homomorphic Encryption*, *IEEE Access*, 10, 117477–117500. doi: 10.1109/ACCESS.2022.3219049
- [8] Joon Woo Lee, Hyungchul Kang, Young Lee, Woojin Choi, Junyeong Eom, Mikhail Deryabin, Eunsang Lee, Junghyun Lee, Donghoon Yoo, Young Sik Kim and Jong Seon No (2022). *Privacy Preserving Machine Learning With Fully Homomorphic Encryption for Deep Neural Network*, *IEEE Access*, 10, 30039–30054. doi: 10.1109/ACCESS.2022.3159694

Operator algebras and modular entropy in holography.

Franco Salinas Pytel, Universidad Complutense de Madrid

Entanglement is one of the defining properties of finite-dimensional quantum systems, often quantified by the von Neumann entropy. In this talk, I will discuss how this notion extends to quantum systems with infinitely many degrees of freedom, as described by quantum field theories (QFT), through the concept of modular entropy. The motivation comes from holographic duality, specifically the AdS/CFT correspondence, which relates the geometry of anti-de Sitter spaces to conformal field theories. This perspective naturally leads to the introduction of Tomita–Takesaki modular theory within the framework of von Neumann algebras. I will conclude by commenting on applications of modular entropy, with particular emphasis on its relation to the computation of Rényi entropies in holography.

References

- [1] Ola Bratteli and Derek W. Robinson, *Operator Algebras and Quantum Statistical Mechanics 1: C^* - and W^* -Algebras. Symmetry Groups. Decomposition of States*, 2nd ed., Springer, Berlin / Heidelberg, 1987.
- [2] Edward Witten, “Notes on Some Entanglement Properties of Quantum Field Theory,” *Rev. Mod. Phys.*, vol. 90, no. 4, Article 045003, 2018; arXiv:1803.04993.
- [3] Daniel Harlow, “TASI Lectures on the Emergence of the Bulk in AdS/CFT,” arXiv:1802.01040 [hep-th], 2018.
- [4] Horacio Casini, Marina Huerta, and Robert C. Myers, “Towards a derivation of holographic entanglement entropy,” *JHEP*, vol. 2011, no. 5, Article 036, 2011; arXiv:1102.0440 [hep-th].

A general theory of nonlocal elasticity based on nonlocal gradients

Guillermo García-Sáez, Universidad de Castilla-La Mancha

Bessel potential spaces have gained renewed interest due to their robust structural properties and applications in fractional partial differential equations (PDEs). These spaces, derived through complex interpolation between Lebesgue and Sobolev spaces, are closely related to the Riesz fractional gradient introduced by Shieh and Spector in [6, 7]. In [3], the equations of nonlocal nonlinear elasticity based on those gradients are studied and related with the well-known Eringen's model. Recently, a broader class of nonlocal gradients have been introduced based on general kernels in [4, 5] that include the particular case of the Riesz fractional gradient. In this talk we present the results obtained in [1, 2] in which we derive the equations of nonlinear elasticity based on the nonlocal gradients for general kernels. Furthermore, we perform a formal linearization of the equations to obtain the linear equations based on those nonlocal gradients. We prove existence and uniqueness of solutions providing a general nonlocal Poincaré and Korn's inequality using a translation operator from the nonlocal Bessel potential spaces to the classical Sobolev spaces. We also study the connection with the Eringen's model in the most general setting and the localization of the equations for varying horizons.

References

- [1] J. C. Bellido, J. Cueto, and G. García-Sáez. Compact embeddings of Bessel potential spaces. Preprint, arXiv:2506.01677, 2025.
- [2] J.C. Bellido and G. García-Sáez *A general theory of nonlocal elasticity based on nonlocal gradients and connections with Eringen's model*, On preparation.
- [3] J. C. Bellido, J. Cueto, and C. Mora-Corral. Eringen's model via linearization of nonlocal hyperelasticity. *Mathematics and Mechanics of Solids*, 29(4):686–703, 2023.
- [4] J. C. Bellido, C. Mora-Corral, and H. Schönberger. Nonlocal gradients: Fundamental theorem of calculus, Poincaré inequalities and embeddings. Preprint, arXiv:2402.16487, 2024.
- [5] J. Cueto, C. Kreisbeck, and H. Schönberger. Γ -convergence involving nonlocal gradients with varying horizon: Recovery of local and fractional models. *Nonlinear Analysis: Real World Applications*, 85:104371, 2025.
- [6] T. Shieh and D.E. Spector. On a new class of fractional partial differential equations I. *Advances in Calculus of Variations*, 8(4):321–336, 2015.
- [7] T. Shieh and D.E. Spector. On a new class of fractional partial differential equations II. *Advances in Calculus of Variations*, 11(3):289–307, 2018.

Balancing Sparsity and Subgroup Fairness in High-Dimensional Regression: The Cost-Sensitive Constrained Elastic Net

Jaime Díaz-Trechuelo Sánchez-Moliní, Universidad de Sevilla

M^a de los Remedios Sillero Denamiel, Universidad de Sevilla, IMUS

The trade-off among predictive accuracy, interpretability, and subgroup fairness is central in high-dimensional regression. A hallmark of regularized regression is the Lasso [1], which achieves variable selection and sparsity but struggles with highly correlated predictors and offers no mechanism for subgroup fairness. Two extensions to the Lasso address these shortcomings separately: the Elastic Net [2], which modifies the objective function by adding an ℓ_2 term to the Lasso's ℓ_1 penalty, ensuring robustness under multicollinearity; and the CSCLasso [3], which modifies the feasible set imposing subgroup-specific bounds on prediction error.

In this talk, we present the Cost-Sensitive Constrained Elastic Net (CSCEN), a novel regularization method developed for my Bachelor thesis that unifies these approaches within a single framework. CSCEN minimizes an Elastic Net-style $(\ell_1 + \ell_2)$ objective *subject to* CSCLasso-style fairness constraints, hence providing a new tool that balances accuracy, interpretability, and fairness through error bounds. By construction, the ℓ_2 term guarantees uniqueness of the solution, while the fairness constraints operationalize subgroup equity.

We examine the mathematical properties of the model, prove uniqueness, and establish conditions for consistency. We then present a series of numerical experiments on real and simulated datasets comparing CSCEN to classical methods, highlighting its ability to achieve sparsity while respecting fairness constraints. This work contributes a flexible and robust framework for modern regression tasks where fairness and structural considerations cannot be sacrificed for predictive accuracy alone.

References

- [1] Tibshirani, R. (1996) “Regression shrinkage and selection via the Lasso”. *Journal of the Royal Statistical Society: Series B (Methodological)*, vol. 58(1): 267-288.
- [2] Zou, H. and Hastie, T. (2005) “Regularization and variable selection via the elastic net”. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, vol. 67(2): 301-320.
- [3] Blanquero, R., Carrizosa, E., Ramírez-Cobo, P. and Sillero-Denamiel, M. R. (2020) “A Cost-Sensitive Constrained Lasso”. *Advances in Data Analysis and Classification*, vol. 15(1): 121-158.

Different landscapes for Phase retrieval

[Jesús Illescas-Fiorito](#), UCM-ICMAT

Phase retrieval concerns the reconstruction of scalar-valued functions from measurements in which only modulus information is available, a problem that arises naturally in many applied mathematics contexts. However, the mathematical formulation of the problem can vary, as can the tools and theories used to study it. As a result, phase retrieval encompasses a wide variety of problems that may appeal to analysts from many areas, including harmonic analysis, complex variables, Banach spaces, Banach lattices, and partial differential equations—a diversity that we aim to illustrate in this poster.

References

- [1] J. Cahill, P. G. Casazza, and I. Daubechies (2016). *Phase retrieval in infinite-dimensional Hilbert spaces*, *Transactions of the American Mathematical Society, Series B*, 3, 63–76.
- [2] D. Freeman, T. Oikhberg, B. Pineau, M. A. Taylor (2023) *Stable phase retrieval in function spaces*, *Mathematische Annalen*, 390(1), 1–43.
- [3] P. Grohs, S. Koppensteiner, and M. Rathmair (2020). *Phase retrieval: uniqueness and stability*, *SIAM Review*, 62(2), 301–350.
- [4] Ph. Jaming (2014). *Uniqueness results in an extension of Pauli’s phase retrieval problem*, *Applied and Computational Harmonic Analysis*, 37, 413–441.

Wedderburn-Artin Theory

Jorge Casanova Moya, Master student at Universidad Complutense de Madrid

A cornerstone of non-commutative algebra is the Artin–Wedderburn theorem, which shows that every semisimple ring is a finite product of matrix rings over division rings. Its proof relies heavily on module theory, a branch of math heavily used by algebraic geometers and illustrates the deep connection between the structure of rings and their representations.

The study of finite-dimensional associative algebras further enriches this perspective. The elegant classification of finite dimensional real division algebras and finite division rings demonstrates how these objects extend familiar number systems.

Tensor products offer a unifying framework, leading to fundamental results such as the Skolem–Noether theorem and the double centralizer theorem. Together, these ideas serve as an invitation to non-commutative algebra, highlighting its elegance, coherence, and far-reaching significance.

References

- [1] Benson Farb and R. Keith Dennis. *Noncommutative Algebra*. Springer, Graduate Texts in Mathematics, 2020. ISBN: 978-1-0716-0893-2. DOI: 10.1007/978-1-0716-0894-9.
- [2] Matej Brešar. *Introduction to Noncommutative Algebra*. Springer, Universitext, 2014. ISBN: 978-3-319-08692-7. DOI: 10.1007/978-3-319-08693-4.
- [3] Matej Brešar. *The Wedderburn-Artin Theorem* [Preprint]. arXiv:2405.04588, 2024. Disponible en: <https://arxiv.org/abs/2405.04588>
- [4] Sharifi, Y. (s.f.), *Noncommutative Ring Theory*. YSharifi's blog. Retrieved from <https://ysharifi.wordpress.com/category/noncommutative-ring-theory/>

Dynamic and numerical methods of average theory with application in oscillator problems

Jorge Rodríguez Pérez, Universidad de Valladolid

This work aims to demonstrate various bounds for the difference between the solution of a system of differential equation where two time-scales are present and its averaged system, using a modern and rigorous approach. The concept of averaging is defined, and properties of KBM functions are analyzed, establishing bounds for the difference between solutions on a time scale of $1/\varepsilon$. Additionally, UKBM functions are introduced to generalize results to unbounded intervals. The theory of averaging for periodic functions is presented in its classical version, studying a change of variables that transforms the original equation into the averaged one, and generalizing to higher-order approximations. Finally, practical applications are presented, such as the analysis of the Kapitza pendulum and the theoretical basis and implementation of the of stroboscopic averaging numerical methods, which allow for the efficient integration of oscillator problems.

References

- [1] Z. Artstein (2007) *Averaging of time-varying differential equations revisited*, Journal of Differential Equations 243.2, pgs. 146–167.
- [2] N. N. Bogolyubov and Y. A. Mitropolsky (1985) *Asymptotic methods in the theory of non-linear oscillations*, Gordon and Breach.
- [3] M. P. Calvo, Ph. Chartier, A. Murua and J. M. Sanz-Serna (2011) *Numerical stroboscopic averaging for ODEs and DAEs*, Applied Numerical Mathematics 61.10, pgs. 1077–1095.
- [4] J. A. Sanders and F. Verhulst (1985) *Averaging methods in nonlinear dynamical systems*, Springer.

Pairs trading strategy based on fractal structures

J. F. Cuevas Rodríguez, University of Almeria, Master's student, Almería, 04120, Spain

M. A. Sánchez Granero, University of Almería, Department of Mathematics, Almería, 04120, Spain

J. F. Gálvez Rodríguez, University of Almería, Department of Mathematics, Almería, 04120, Spain

Delta-neutral trading strategies aim to construct portfolios with minimal exposure to market volatility, keeping their value stable regardless of directional movements in the underlying assets. Building on the Fractal Market Hypothesis, we assume that the valuation functions of financial assets trace fractal curves, making them suitable for analysis through fractal structure theory.

In this poster, we focus on a pairs trading strategy. The central idea is to identify pairs of assets whose prices display strong and persistent correlation. Under the assumption that this relationship is long-lasting, we exploit mean-reversion signals to generate consistent returns.

Within the framework of fractal structure theory, we define a fractal structure over the studied time horizon and develop a fractal approximation of each asset's valuation function as a function of time. This allows us to leverage the underlying properties of the Fractal Market Hypothesis. Applied to real data, this strategy yields an annualized return of 8.6% over the 2019-2024 period, based on asset pairs selected from the Standard & Poor's 500 index.

References

- [1] F. G. Arenas, M. A. Sánchez-Granero. Hahn-Mazurkiewicz revisited: A new proof. *Houston J. Math.*, **28**(4):753–769, 2002.
- [2] J. Blackledge, M. Lamphiere. A review of the fractal market hypothesis for trading and market price prediction. *Mathematics*, **10**(1):117, 2022. <https://doi.org/10.3390/math10010117>
- [3] M. Fernández-Martínez, M. A. Sánchez-Granero. Fractal dimension for fractal structures. *Topology Appl.*, **163**:93–111, 2014.
- [4] M. Fernández-Martínez, M. A. Sánchez-Granero. A new fractal dimension for curves based on fractal structures. *Topology Appl.*, **203**:108–124, 2016.

Introduction to the compact Lie groups representation theory

Julia Reina Martín, Universidad de Málaga

The main objective of this work is to demonstrate that any representation of a compact Lie group over a finite-dimensional $\mathbb{K}$ -vector space, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, is entirely determined by its character, that is, by a differentiable function.

To this end, firstly, the concepts of a representation of a Lie group G on a finite-dimensional vector space V , $\rho : G \rightarrow \text{Aut}(V)$, $g \mapsto \rho(g) \equiv \rho_g$, and of a G -linear map are defined. Furthermore, an equivalence relation is established on the set of representations of the group based on the existence of a G -linear isomorphism.

Secondly, some examples of constructing new representations from given ones will be shown, leading to the definition of a *reducible* or *irreducible* representation. In addition, *Schur's Lemma*, an important result for achieving our objective, will be stated and proved.

On the other hand, the concept of a *completely reducible representation* is defined, and it is demonstrated that all representations of a compact Lie group are completely reducible. For this purpose, it is necessary to define the concept of a G -invariant inner product.

Finally, the *character* of a representation is defined, which is the differentiable function $\chi_V : G \rightarrow \mathbb{K}$, $\chi_V(g) = \text{trace}(\rho_g)$. Some properties of this map will be shown, and the set of fixed points of the representation, V^G is defined, also proving the following equality: $V^G = \{\int_G gv \, dg : v \in V\}$. To conclude, an inner product is defined in the vector space generated by the characters of all the representations of G , this final step allowing for the demonstration of our objective: any representation of a compact Lie group is completely determined by its character.

References

- [1] Theodor Bröcker y Tammo Tom Dieck (2003) *Representations of Compact Lie Groups*, Springer Science and Business Media.
- [2] William Fulton y Joe Harris (2004) *Representation Theory: A First Course*, Springer Science and Business Media.
- [3] Brian C. Hall (2015) *Lie Groups, Lie Algebras, and Representations*, Springer, 2nd ed.

Mathematical anxiety and its assessment: visibility, measurement scales and educational challenges

[Laura Barrera-Romero](#), Loyola University

Carlos Fresneda-Portillo, Loyola University

Salvador Reyes-de-Cózar, Loyola University

Math anxiety is a complex phenomenon that affects students of different ages and educational levels, limiting their performance, confidence, motivation, and even future career choices, particularly in STEM fields. Despite its relevance, it has often been overlooked in education, reduced to a lack of skill or effort [1]. However, research shows that it is a multidimensional construct requiring recognition within the educational framework. This study conducted a systematic review using the PRISMA methodology, examining measurement scales and assessed factors to determine whether there is consensus on math anxiety and whether there is a reliable scale.

A central focus has been on the creation and adaptation of questionnaires for a more rigorous understanding. The literature includes numerous validated tools for different populations, contexts, and ages. Among the best known is the Mathematics Anxiety Rating Scale (MARS) by Richardson & Suinn (1972) and its abbreviated versions. Specific scales for children and adolescents [2] reflect both the complexity of the phenomenon and the need for instruments adapted to different realities.

The results show that although there is some agreement on relevant factors, the scales vary in focus. In education, reliable data on students' anxiety levels enable inclusive strategies, targeted interventions, and a change in the social perception of mathematics. Addressing math anxiety thus represents both a challenge and an opportunity to foster equitable learning and a healthier relationship with this discipline.

Key words: mathematical anxiety, dimensions, measurement, educative context.

References

- [1] Maldonado, M., & Sotomayor, V. (2021). Análisis de la ansiedad matemática en futuros emprendedores - [Analysis of math anxiety in future entrepreneurs]. *Innovaciones docentes en tiempos de pandemia*.
- [2] González Medina, M. A., & Treviño Villarreal, D. C. (2024). Ansiedad hacia las matemáticas en bachillerato: una relación con el fomento del razonamiento, pensamiento y el apoyo docente. *European Public & Social Innovation Review*, 9, 1–16. <https://doi.org/10.31637/epsir-2024-305>

Dirichlet's Theorem: Primes in arithmetic progressions

Lorenzo María López Domene, University of Alicante

In 1837, Dirichlet proved the existence of infinitely many primes in arithmetic progressions $\{a + nq\}_n$ with $\gcd(a, q) = 1$, thereby confirming a conjecture of Legendre and inaugurating analytic number theory. In this talk, we will introduce the central objects involved in Dirichlet's argument and present a streamlined variation of his original proof, which preserves its essential ideas while making them more transparent.

References

- [Apo76] T. M. Apostol. *Analytic number theory*. Undergraduate Texts in Mathematics. Springer-Verlag, New York-Heidelberg, 1976.
- [Dav00] H. Davenport. *Multiplicative number theory*. Second edition. Graduate Texts in Mathematics, Vol. 74. Springer-Verlag, New York, 2000.
- [Ser73] J. P. Serre. *A course in arithmetic*. Graduate Texts in Mathematics, Vol. 7. Springer-Verlag, New York-Heidelberg, 1973.

Sheaf and de Rham Cohomologies: The de Rham Theorem

Malena Domínguez Sirgo, Universidad Complutense de Madrid

This work studies the relationship between de Rham cohomology and sheaf cohomology, two central tools in modern geometry and topology. Starting with a detailed introduction to the concept of sheaves, using the étale space approach, we cover basic aspects of the theory of sheaf cohomology, proving that all sheaf cohomology theories for the same space are equivalent.

We then present de Rham cohomology, along with its classical properties. Finally, we obtain the desired result connecting both theories: the de Rham Theorem, which together with the previously mentioned result establishes an isomorphism between de Rham cohomology and sheaf cohomology with coefficients in the constant sheaf. This theorem not only highlights the deep relationship between the two theories but also shows how each framework can provide insight into the other. To make this connection concrete, we finish this work by including explicit computations of de Rham cohomology for the torus and the sphere, demonstrating how these results translate into sheaf cohomology.

References

- [1] Dennis Barden and Charles B. Thomas (2003) *An Introduction to Differential Manifolds*, Imperial College Press
- [2] Frank W. Warner (1983) *Foundations of Differentiable Manifolds and Lie Groups*, Springer-Verlag

Robust probability support vector machines with pinball loss

Manuel Gonzalo Carvajal ^a,

Miguel Carrasco ^b, Benjamín Ivorra ^a, Julio López ^c, Angel M. Ramos ^a,

^a, Instituto de Matemática Interdisciplinar, Departamento de Análisis y Matemática Aplicada,
Universidad Complutense de Madrid.

^b, Facultad de Ingeniería y Ciencias Aplicadas, Universidad de los Andes.

^c, Facultad de Ingeniería y Ciencias, Universidad Diego Portales.

Support Vector Machines, SVMs for short, are Machine Learning algorithms mainly used to solve the problem of binary classification. These algorithms work by constructing a classifier function from a set of previously labeled data by solving a convex optimization problem.

Although neural networks are more versatile and effective in many supervised learning applications, they are also prone to overfitting and classifying new observations can have a high computational cost. This makes SVMs more suitable for small databases or deployment in smaller devices.

These advantages have motivated the research in SVM type models as an alternative to neural networks in certain situations. In this context several modifications have been proposed to improve the performance of these algorithms. In our work we focus on two adaptations. First, the probability SVMs presented in [2], that produce a probability as the output instead of just the classification. Second, the utilization of the pinball loss function as suggested by [3], that results in models that are more robust to noise.

In our work we propose a novel SVM model that combines both modifications and obtain the primal and dual form of the corresponding optimization problem. Then we implemented the problem and tested its performance over several databases focusing on the robustness on the solution to perturbation on the input data.

References

- [1] I. Steinwart and A. Christmann (2008) *Support Vector Machines*, Springer, New York.
- [2] Yuan-Hai Shao, Xiao-Jing Lv, Ling-Wei Huang and Lan Bai (2023) *Twin SVM for conditional probability estimation in binary and multiclass classification*, Pattern Recognition.
- [3] Xialoin Huang, Lei Shi and Johan A. K. Suykens (2014) *Support Vector Machine Classifier With Pinball Loss*, IEEE Transactions on Pattern Analysis and Machine Intelligence.
- [4] Miguel Carrasco, Benjamín Ivorra, Julio López and Angel M. Ramos (2025) *Embedded feature selection for robust probability learning machines*, Pattern Recognition.

An Introduction to Nonstandard Analysis: Where Infinitesimals Truly Exist

Marc Ventura, Universitat de València

The use of infinitesimal and infinite numbers in mathematics dates back to the origins of calculus. It was not until the 1960s that Robinson [1] provided a rigorous foundation for these ideas through Nonstandard Analysis (NSA). Instead of treating infinitesimals as vague notions, NSA constructs a precise number system extending the real numbers, known as *hyperreal numbers*. While often associated with model theory, there is a more intuitive algebraic approach: the hyperreal numbers can be built explicitly using ultraproducts (see [2]). This talk provides an accessible introduction to NSA for functions of a single real variable, covering constructions, the extension of sets and functions, and the characterization of limits and derivatives through infinitesimals. A central feature of NSA is the *Transfer Principle*, which allows many properties of the real numbers to carry over naturally to the hyperreals. We will also briefly discuss, with examples, how nonstandard methods can be applied in advanced contexts to prove new results or simplify existing proofs.

References

- [1] Abraham Robinson (1966) *Non-standard analysis*, North-Holland Publishing, New York.
- [2] Robert Goldblatt (1998) *Lectures on the Hyperreals. An Introduction to Nonstandard Analysis*, Springer-Verlag, New York.

G-kernels on Operator Algebras

Marina Polo Rodríguez, KU Leuven

This poster presents key ideas from my Master's thesis on the theory of G -kernels on operator algebras. A G -kernel is defined as a group homomorphism $\alpha : G \rightarrow \text{Out}(A) = \text{Aut}(A)/\text{Inn}(A)$ from a group G into the outer automorphism group of a C^* -algebra A . The gauge symmetries of a system, which are considered redundant from the perspective of observable physical properties, correspond to the inner automorphisms of the algebra describing the system. Consequently, the elements of $\text{Out}(A)$ represent the non-redundant symmetries of the system, making the study of the outer automorphism group crucial for physics.

Every G -kernel is associated with a crucial cohomological invariant, its lifting obstruction, an element in $H^3(G, U(Z(A)))$. A central focus is the realization problem: given a finite group G and a cohomology class in $H^3(G, \mathbb{T})$, can one construct a G -kernel on a UHF-algebra (inductive limits of matrix algebras) with this obstruction?

We explore Izumi's conjecture, which provides a necessary and sufficient condition for such realizations in terms of the order of the obstruction and the supernatural number defining the UHF-algebra. In particular, we outline the proof of the conjecture for finite abelian groups.

Additionally, we discuss how the existing constructions that realize these kernels, such as Connes' construction for cyclic groups, can be understood as AF-actions, which are actions defined via compatible sequences on finite-dimensional algebras.

References

- [1] Kenneth S. Brown. *Cohomology of groups*. Vol. 87. Graduate Texts in Mathematics. Springer-Verlag, New York-Berlin, 1982.
- [2] A. Connes. "Periodic automorphisms of the hyperfinite factor of type II_1 ". In: *Acta Sci. Math. (Szeged)* 39.1-2 (1977), pp. 36-66.
- [3] S. Girón Pacheco. "Anomalous symmetries of classifiable C^* -algebras". PhD thesis. University of Oxford, 2023.
- [4] Masaki Izumi. *G-kernels of Kirchberg algebras*. 2024. arXiv: 2309.03441
- [5] V. F. R. Jones. "An invariant for group actions". In: *Algèbres d'opérateurs (Sém., Les Plans-sur-Bex, 1978)*. Vol 725. Lecture Notes in Math. Springer, Berlin, 1979, pp. 237-253.

Conjugacy of parabolic subgroups in Dyer groups

María Cumplido, Universidad de Sevilla

[Marina Salamero](#), Universidad de Sevilla

Mireille Soergel, TU Berlin

Dyer groups form a family that generalizes Coxeter groups, RAAGs (Right-Angled Artin Groups) and graph products of finite cyclic groups. They admit a uniform solution to the word problem and allow the definition of parabolic subgroups in a manner analogous to that of Coxeter and Artin groups. These subgroups play a fundamental role in the study of the topological and algebraic properties of said groups. In 1997, Luis Paris, building on the work of Kramer for Coxeter groups, proposed an algorithm that efficiently determines whether two parabolic subgroups are conjugate in the Artin group. In this poster, we will present an algorithm, based on the works of Paris and Kramer, that decides whether two parabolic subgroups of a Dyer group are conjugate.

References

- [1] D. Krammer, *The Conjugacy Problem for Coxeter Groups*, PhD thesis, Utrecht (1994)
- [2] L. Paris, *Parabolic subgroups of Artin groups*, J. Algebra, 196 (1997)
- [3] L. Paris and M. Soergel, *Word problem and parabolic subgroups in Dyer groups*, Bull. Lond. Math. Soc., 55 (2023)

Control theory in finite-dimensional spaces

First author, Miguel Trujillo Alés.

Tutor bachelor thesis, Diego Araujo de Souza.

The aim of this work is to give an introduction to control and stabilization theory in finite-dimensional spaces. It will be started exposing the basic concepts of controllability in finite-dimensional spaces. Throughout this first part, it is intended to apply these controllability concepts to different types of control systems. First, it will be studied the controllability of linear systems, for both autonomous and non-autonomous cases, with and without control constraints. It will be also seen the controllability of non-linear systems, but only local results. The main idea will be to establish sufficient conditions that ensure the controllability of nonlinear control systems. Finally, it will be also exposed the basic concepts of stability/stabilization in finite-dimensional spaces. Firstly, it is intended to apply these concepts to autonomous linear systems and it will be seen the difficulties that emerge when one tries to stabilize non-autonomous linear systems. It will be also studied the stabilization of non-linear control systems, where the theory of Lyapunov functions will be useful.

References

- [1] E. Trélat, *Contrôle optimal. Théorie & applications. Mathématiques Concrètes. (Français)* [*Control óptimo. Teoría y aplicaciones. Matemáticas concretas*], Vuibert, Paris, 2005, vi+246 pp.
- [2] E.D. Sontag, *Mathematical control theory. Deterministic finite-dimensional systems. (Inglés)* [*Teoría de control matemático. Sistemas determinísticos finito-dimensionales*] Segunda edición, Textos en Matemáticas Aplicadas, 6, Springer-Verlag, New York, 1998, xvi+531
- [3] J-M. Coron. *Control and nonlinearity. (Inglés)* [*Control y no-linealidad*], Mathematical surveys and monographs, 1956, ISSN 0076-5376 ; v. 136.
- [4] D. Hinrichsen, A. J. Pritchard. *Mathematical Systems Theory I, Modelling, State Space Analysis, Stability and Robustness (Texts in Applied Mathematics). (Inglés)* [*Teoría de sistemas matemáticos I, modelado , análisis de espacio de estados y robustez (Textos en matemáticas aplicadas)*]
- [5] M. Bodson. *Explaining the Routh-Hurwitz criterion. A tutorial presentation (Inglés)* [*Explicación del criterio de Routh-Hurwitz. Un tutorial de presentación*], 15 de septiembre de 2019.
- [6] Hassan K.Khalil, *Nonlinear systems. (Inglés)* [*Sistemas no lineales*]. Prentice Hall, 2002.

Morse theory and 2D stationary Euler flows

Natalia Averno García, Universidad Complutense de Madrid (UCM)

This work characterizes steady solutions of the 2D Euler equation for ideal fluids on surfaces with possible boundary, employing a geometric and combinatorial framework rooted in differential topology, specifically, Morse theory. Motivated by Arnold's Hamiltonian formulation of hydrodynamics, we classify Morse-type coadjoint orbits (those whose vorticity is a simple Morse function) using circulation Reeb graphs. These graphs provide a complete invariant for coadjoint orbits under symplectomorphisms.

The central result establishes a sharp criterion for the existence of steady flows: a coadjoint orbit admits a steady solution if and only if its circulation function is balanced, a condition enforcing consistent signs near critical vorticity levels. This transforms the dynamical problem into a combinatorial constraint on the Reeb graph. For closed surfaces, the balanced condition simplifies further to total negativity of the circulation function, revealing that steady flows are rare and topologically constrained. Explicit examples on genus-two surfaces illustrate how the solution space forms a convex polytope within the space of circulations. The work also includes an application example of the main result, constructing a set of curves as streamlines of a steady flow on the sphere.

By bridging fluid dynamics, symplectic geometry, and topology, this work studies new insights into the interplay between vorticity and stability in ideal hydrodynamics.

References

- [1] V.I. Arnold y Boris A. Khesin (2021) *Topological methods in hydrodynamics*, Springer, Cham, Switzerland, 2nd ed., Applied Mathematical Sciences, 125. ISBN: 3-030-74278-4.
- [2] A. Izosimov y B. Khesin (2016) "Characterization of Steady Solutions to the 2D Euler Equation", *International Mathematics Research Notices*, vol. 2017, no. 24, pp. 7459–7503. DOI: 10.1093/imrn/rnw230.

Online Follow-The-Leader (FTL) Model Selection with Conformal Prediction

Nuria Richer Gusano, BCAM – Basque Center for Applied Mathematics

We benchmark hourly probabilistic forecasts selected online by a Follow-The-Leader (FTL) rule over a grid of smoothing and regularization parameters (λ_s, λ_r) . Three selection metrics are compared—RMSE (point error), CRPS (distributional accuracy), and NLL (likelihood fit)—against a baseline model across four datasets. RMSE emphasizes point accuracy; CRPS balances sharpness and calibration; NLL rewards calibrated, often wider, predictive densities. Evaluation uses RMSE, MAPE, CRPS, NLL, and Pinball, with calibration diagnostics from PIT histograms and ECDFs.

Beyond this, we extend the framework with *conformal prediction* to provide finite-sample, distribution-free coverage guarantees on the forecast intervals, ensuring reliability under model misspecification. We also incorporate *dynamic learning* updates, allowing the selection mechanism to adapt not only to cumulative past losses but also to regime shifts and non-stationarities in real time. These enhancements complement FTL by strengthening robustness: conformal layers control coverage, while dynamic adaptation accelerates responsiveness to changing data patterns.

Results show that FTL–CRPS consistently achieves the best probabilistic accuracy (lowest CRPS and Pinball), while FTL–NLL optimizes tail safety (best NLL). Point accuracy remains dataset-dependent. The combined framework highlights a practical trade-off: CRPS for reliable quantiles, NLL for tail robustness, and conformal + dynamic extensions for adaptive, trustworthy deployment in evolving environments.

References

- [1] Gneiting, T., & Raftery, A. E. (2007). *Strictly proper scoring rules, prediction, and estimation*. Journal of the American Statistical Association, 102(477), 359–378.
- [2] Vovk, V., Gammerman, A., & Shafer, G. (2005). *Algorithmic learning in a random world*. Springer.
- [3] Cesa-Bianchi, N., & Lugosi, G. (2006). *Prediction, learning, and games*. Cambridge University Press.

Semi-Lagrangian Approach to Kinetic Models in Plasma Simulations

Pietro Nardelli, University of Ferrara

Plasma is a gaseous state of matter consisting of charged particles, specifically electrons and ionised atoms. In modern applied research, several lines of investigation focus on plasma modelling, ranging from nuclear fusion to medicine and astrophysics. In this context, reliable numerical methods are needed to provide accurate simulations. Due to the presence of ions, the motion of plasma particles is governed by electromagnetic forces; moreover, they react to external electric or magnetic fields, changing the overall system behaviour. This complex behaviour makes plasma challenging to simulate efficiently.

In this work, we illustrate one of the methods employed in plasma simulations: the *semi-Lagrangian approach*. The method relies on mathematical tools such as operator splitting [5] and polynomial interpolation [4], and we present it here in the case of the *one-dimensional Vlasov-Poisson system* [1], one of the most studied plasma models. Lastly, we assess the performance of the semi-Lagrangian method with some standard plasma physics test cases, specifically Landau damping and the two-stream instability [3][2].

References

- [1] Chio, Z.C. and Knorr, G. (1976) *The integration of the Vlasov equation in configuration space*, Journal of Computational Physics, Volume 22.
- [2] Mulet, P. and Vecil, F. (2012), *A semi-Lagrangian AMR scheme for 2D transport problems in conservation form*, Journal of Computational Physics.
- [3] Sandberg, R.T., Krasny, R. and Thomas, A.G.R. (2024) *The FARSIGHT Vlasov-Poisson code*, Journal of Computational Physics.
- [4] de Boor, C. (1980), *A Practical Guide to Splines*, McGraw-Hill.
- [5] Strang, G. and MacNavara, S. (2016), *Splitting Methods in Communication, Imaging, Science, and Engineering*, Springer International Publishing Switzerland.

Organizing committee

- Ana Casado Sánchez (US)
- Javier Díaz Cabrera (US)
- Laura García Rastrojo (US)
- Daniel Gómez Gutiérrez (US)
- Beatriz Marín Gimeno (UMU)
- Alfonso Márquez Martínez (US)
- Ángel Méndez Vázquez (USC)
- Miguel Navarro Castro (US)
- Javier Polo Noche (USC)
- Pablo Regalado García (US)
- Marina Salamero Cebollero (US)
- Julia Sánchez Loscertales (UZ)

Scientific committee

- Alejandro Bandera Moreno (ULo)
- Beatriz Barbero Lucas (UCD)
- Silvia Bordel Vozmediano (UCLM)
- Víctor Carmona Sánchez (MPI-MiS)
- Francisco José Cruz Zamorano (ULL)
- Antonio Hidalgo Torné (MPI-MiS)
- Juan Manuel Lorenzo Naveiro (OU)
- Alejandro Mahillo Cazorla (UV)
- Ana Mayora Cebollero (UZ)
- Garazi Retegui Goñi (UPNA)
- Juan Miguel Ribera Puchades (UIB)