Planning transportation networks through optimization models

Course 4: Modelling and optimization algorithms in networks design and energy planning

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Komsomólskaya Metro Station (Moscow)  
Canary Wharf Metro Station (London)
Outline

1. An Overview of Transportation Planning Problems

2. Parameter Analysis in the Design of Rapid Transit Networks with Transfers.

3. Rescheduling railway timetables in presence of passenger transfers between lines within a transportation network.

1. An Overview of Transportation Planning Problems
Operations Research

• Started during World War II
• Applied since the 1950s to several production and manufacturing problems
• Major applications since the 1970s to the field of transportation
• Increasing since the 1980s with the advent of new technologies

George Bernard Dantzig (1914-2005)
The application of OR to transportation is a success story
• Economic importance of transportation
• Structure of transportation problems
• Large scale
• Large volumes

Objective of this introduction
• To provide an overview of the main transportation problems
• To illustrate the application of optimization techniques of the field of transportation

Outline
• How is the field of transportation structured?
  – Transportation modes
  – Planning levels
  – Availability of information
• Examples of successful OR applications
• Assessment and research prospects
How is the field of transportation structured? (i)

Transportation modes

- **Air**: Passengers, cargo, taxi planes.
- **Rail**: Passengers, freight, suburban, long-distance.
- **Truck**: Short-haul, long-haul, hazmat transportation.
- **Ship**: Lines, coastal traffic, ferries, tankers.
- **Public transit**: Buses, metro, suburban trains.
- **Car**
- **Taxi**: Taxi fleets, dial-a-ride services.
- **Bicycle**
- **Pedestrian**: Flows, evacuation routes.
- **Emergency vehicles**: Ambulances, fire trucks, police cars
- **Public service vehicles**: Snow removal, street cleaning, garbage collection
- **Automated guided vehicles**: Path location, operations
- **Pipelines**
- **Space travel**: Satellites, space missions
- **Intermodal**: Bus-train, ship-truck (containers), truck-train (containers)
- **Etc.**
How is the field of transportation structured? (ii)

Planning levels (examples)
• **Strategic** (several years)
  – Network design and location of major facilities: airports, motorways, metro lines
  – Fleet acquisition
• **Tactical** (several weeks or months, a few years)
  – Route choices, bus lines
  – Markets
  – Location of intermediate facilities: depots, warehouses
  – Design of seasonal routes, vehicle scheduling, staff scheduling
  – Pricing
• **Operational** (several hours or days)
  – Vehicle routing
  – Container movements
  – Landing slots allocation
  – Recovery operations
  – Dial-a-ride operations
• **Real-time** (seconds or minutes)
  – Courier services
  – Ambulance location and relocation
  – Fire truck assignment
How is the field of transportation structured? (ii- bis)

Planning levels (examples)
- Strategic
- Tactical
- Operational
- Real-time
How is the field of transportation structured? (iii)

Availability of information
• Static problems
  – All information is available at the time of planning (e.g., bus route design).
• Stochastic problems
  – Probabilistic information is available at the time of planning
  – First stage solution is planned
  – Random event occurs
  – Recourse action is taken (second stage solution) (e.g., airline rescheduling).
• Dynamic problems
  – No a priori information is available
  – Information is gradually revealed over time (e.g., taxi or ambulance operations, dial-a-ride services, courier services).
Examples of successful OR applications (i)

... Rail transportation

**Passenger trains**
- network design
- train schedules
- train blocking
- crew pairing
- several of these problems are solved through ILP by means of decomposition techniques, network flow algorithms, etc.

**Freight trains**
- very fragmented industry in North America
- not as well organized and optimized as the airline industry
- gains can be made through better crew scheduling, train blocking and routing, pricing policies
- used to poor service standards

**Suburban trains**
- almost run like buses

Transportation Management reference:
Examples of successful OR applications (ii)

Passenger Trains
– More and more presence in Europe.
– Management of infrastructure: responsibility of Governments (Infrastructure Manager).
– Operating trains: responsibility of Train Operators.

In practice,
• Train Operators provide their preferred timetable, rolling stock and transport services.
• Infrastructure Manager is responsible for train planning and real-time traffic control.

Line planning
– Line: subset of trains having same route and same set of stops (only their schedules differ).
– Line planning problem: design a line system such that all travel demands are met, passenger service is maximized and operational costs are minimized.
– This includes assigning the types of trains to the lines and determining the services.
– Service level may also influence demand (induced demand).
**Examples of successful OR applications (iii)**

**Train timetabling**
- Cyclic timetables (e.g. xx12 and xx42) are easy to remember but costly.
- Do not adjust to demand (except that longer trains may be provided at peak hours).
- Non-cyclic timetables: relevant for heavy-traffic and long-distance corridors.
- Each Train Operator associates a priority to each train, an ideal timetable and tolerances.
- The Infrastructure Manager minimizes deviations between ideal and actual timetables.

**Train platforming**
- Assignment of trains to platforms in stations.
- Minimum elapsed time between departure of last train and arrival of next train.
Examples of successful OR applications (iv)

Rolling stock circulation
• Decide on type and number of rolling stock units for each train.
• Composition of trains may be modified at some stations.
• Determine the number of locomotives for each train (sometimes to allow movement in both directions).
• Determine maintenance schedules.
• Reservation systems or not (typically no reservations in dense networks or for commuter trains).

Train unit shunting
• During the night trains are parked in shunting areas.
• Determine where to park the trains in order to achieve smooth operations.

Crew planning
• Construct work schedules for trains’ employees.
• Crew scheduling: determine the number and types of employees needed for all trips; determine duties, i.e. sequences of trips covered by a single crew over one or two days.
• Crew rostering: sequence duties to obtain rosters.
Examples of successful OR applications (v)

Public transit
- Design of metro lines
  - ensure good trip or population coverage
  - maximize population mobility
  - area in which it is difficult to optimize
- Bus transportation
  - network design (bus routes): O/D surveys
  - bus schedules: ensure fit with demand
  - bus assignment to routes
  - drivers assignment to bus schedules and buses in order to
  minimize “extra” costs (similar to airline crew pairing)
Examples of successful OR applications (vi)

Operations research techniques used in transportation
• Exact optimization methods: large scale integer programs solved by decomposition:
  – constraint relaxation, Lagrangean relaxation
  – column generation
  – Benders decomposition
  – network flow techniques
• Heuristics
  – simple tricks: insertion, exchange methods, etc.
  – metaheuristics: simulated annealing, tabu search, genetic and population search, ant systems, neural networks, etc.
• Forecasting
• Simulation
• Queueing theory
• Decision trees
• Etc.
Assessment

Positive elements
- Problems are important but they are well structured
- Optimization techniques are now very powerful
- Better software is available (CPLEX)
- Technology is improving (computers, GIS, user interfaces, data management techniques, GPS, on-board computers).

Negative elements
- Systems are complex
- Stochasticity
- Forecasting is difficult
- Economies of scale do not always occur
- Pricing is difficult
- Resistance to change
- Politics

Research prospects
- Real-time problems
- Pricing
- Integrated models
- Robustness
- Improving algorithms
  - accuracy  - speed  - simplicity  - flexibility
2. Parameter Analysis in the Design of Rapid Transit Networks with Transfers
Rapid Transit Systems (RTS)

- Motivation for building RTS in cities:
  - Increasing mobility
  - Enlargement of the urbanized areas
  - Reduction of average surface traffic speed
  - Sustainability, etc.

→ Planning RTS must adequately solve two intertwined problems which take part in the design process:

  - Determination of alignments
  - Location of stations.
THE STRUCTURE THAT SUPPORTS THE TRANSIT LINES IN TRANSPORT NETWORK

METRO NETWORK

LINE NETWORK

INFRASTRUCTURE FOR THE CONNECTIONS
The RTS design problem involves at upper level the node and edge locations and at lower level, the user traffic behavior.

At upper level
1. Objective: Maximizing Trip Coverage of Public Mode
2. Own constraints and budget constraints

At lower level
1. Customers choose the most convenient paths (direct trips or with few transfers)
2. Customers compare private and public trip costs

The trip demand, combined with the earlier facts, leads to different topological configurations when the RTS is designed.
SOME SIMPLE TOPOLOGICAL PATTERNS

(i)

Cartwheel
Star
U and Cross
Circumferential
Grid
Triangle

(Laporte et al., )
SOME SIMPLE TOPOLOGICAL PATTERNS (ii)

The Prague underground railway network

Triangle
Using optimization techniques

- The use of Optimization Techniques has been directed towards
  - Determining a single alignment
    Maximizing the estimated coverage (in terms of population near to station)
      * Dufourd, Gendreau and Laporte (1996)
      * Bruno, Gendreau and Laporte (2002)
    Maximizing coverage calculated with OD data
      * Laporte, Mesa and Ortega (2003)
  
  - Locating stations once the alignment is given
    * Hamacher, Liebers, Schöbel, Wagner and Wagner (2001)
    * Laporte, Mesa and Ortega (2002)
  
  - An integrated methodology (alignments+stations)
    * Laporte, Marín, Mesa, Ortega and Sevillano (2005)
      + Designing networks in regard to transfers
        * Garzón-Astolfi, Mesa and Ortega (2005)
Population covered by the RTS

• In the first approaches for determining an optimal alignment (a network composed of a single line), the covered population was estimated proportionally to the number of people living within a certain distance from the corridor.


• The weakness of this method was that it did not take into account that people living near the line, but relatively far from a station, were less likely to use the network.
A better way to measure the demand is to compute only the population covered by stations. In


Isochronous curves around each potential station are used, with weights $\theta_d$ applied to the population living within $d$ units of distance (walking) from the station. They consider four levels of attraction.

Walking distances can be approximated by a metric $l_p$, where usually $1 \leq p \leq 2$.

$$l_p(x, y) = \sqrt[p]{x^p + y^p} = 1$$
Covered population by the RTS

• Let $\Pi(x, y)$ be the population associated with the coordinates of the point $(x, y)$.
• Coverage due to the station located at $s$ is defined by

$$C(s) = \sum_d \sum_{(x, y):D[(x, y),s]=d} \theta_d \Pi(x, y)$$

• Hence, coverage $C(P)$ due to the whole alignment $P=(s_1, ..., s_n)$ is

$$C(P) = \sum_{i=1}^{n} C(s_i)$$
The notion of trip coverage, introduced by Mesa and Ortega (2001), combines the estimation of population captured at the stations, according to some model of gravitational attraction, and the users' trip preferences that provide the OD matrices.

\[ f_{ij} = \sum_{k,k'} OD_{kk'}(i, j) \]

\[ OD_{ij}(k,k') = \sum_{l,m=1,(l \neq m)}^L \frac{a^2}{(r_k)^2 \cdot (r_{k'})^2} \cdot \frac{\text{Area}(B_{ik} \setminus B_{i(k-1)}) \cap z_l}{\text{Area}(z_l)} \cdot \frac{\text{Area}(B_{i'k'} \setminus B_{i'(k'-1)}) \cap z_m}{\text{Area}(z_m)} \cdot t_{lm} \]

This measure is more accurate than C (P) but requires a considerable amount of data and is more difficult to handle within an optimization algorithm.
**Integrated methodology (i)**

- Once decided the methodology for calculating the population served, we can establish a design procedure based on two stages.

- **FIRST STAGE:** Select a small number of sites that generate a high number of trips and Formulate an optimization model for designing the RTS.

  1. **Attractive locations:** provide facilities or a high number of jobs
     - office zones
     - industrial areas
     - commercial zones
     - universities
     - hospitals, ...

  2. **Trip generators:** points of multimodal interchange (railway and bus stations, airports, …) and densely populated areas which are situated far away from the central area of the city.

  3. **Park and Ride facilities:** points on the access roads to the city with the highest inflow.
Integrated methodology (ii)

- SECOND STAGE: Determine other intermediate stations on the initial arcs along the transit lines of the network.
  * Hamacher, Liebers, Schöbel, Wagner and Wagner (2001)
  * Laporte, Mesa and Ortega (2002)

- Determining the best locations for building stations along a corridor
- Candidate points: 9
An instance: Metropolitan area of Sevilla
Key nodes and main corridors
Population density
Flows between adjacent sectors
ACCESS TO SEVILLA: GENERATED / ATTRACTED TRIPS
Main corridors
A solution based on our integrated methodology
A different solution based on the corridor analysis
Complete Graph

A line in the graph

**Adjacency matrix**

<table>
<thead>
<tr>
<th>OD</th>
<th>1</th>
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<th>3</th>
<th>4</th>
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</table>
Integrated methodology (ii)

• Let $N = \{n_i : i = 1, 2, \ldots, I\}$ be a set of known potential locations for key stations. Typically for a medium size agglomeration: $5 \leq I \leq 20$

• Let $E$ be the set of feasible edges linking the key stations.

\[ \rightarrow \text{Select the core network in } G = (N, E) \]
Integrated methodology (iii)

• We suppose that the travel patterns are given by an OD matrix

\[ OD = \left\{ g_p : p = (q, r) \in P \right\} \]

where \( P \) is the demand ordered pair set.

\[ D = \left\{ d_{qr} \right\} \]

• A matrix of distances among the points in \( N \) is given.

→ The network design model is formulated in terms of integer programming and is addressed to the maximization of the trip coverage which provides the rapid transit system to the users, taking relevant costs and the competition between public and private modes of transportation into account.
The cost structure

• Let $c_{ij}$ and $c_i$ denote the costs of constructing a section of an alignment in edge $(i,j)$ and that of constructing a station at node $n_i$.

• The user generalized cost of satisfying the demand of pair $p$ through the private and the public network are respectively $uc_{p}^{PRIV}$ and $uc_{p}^{PUB}$.

• Note that $uc_{p}^{PUB}$ depends on the final topology of the public network and therefore on the edges that are selected.

• Moreover, the following bounds are known

$$\text{length}_{min}^{l}, \text{length}_{max}^{l}, \text{Tlength}_{min}, \text{Tlength}_{max}$$

Select a low number $L$ of lines (typically, $2 \leq L \leq 5$) covering as much as possible the travel demand between the points of $N$, subject to the cost constraints.
The variables of the model (i)

- Station selection:
  \[ y^l_i = 1 \text{ if } n_i \in A_i; \quad y^l_i = 0, \text{ otherwise.} \]

- Edge selection:
  \[ x^l_{ij} = 1 \text{ if edge } ij \in E \text{ is selected for line } l; \quad x^l_{ij} = 0, \text{ otherwise.} \]

- Demand selection:
  \[ u^p_{ij} = 1 \text{ if demand of pair } p \text{ would use edge } ij \in E; \quad u^p_{ij} = 0, \text{ otherwise.} \]
The variables of the model (ii)

- Mode choice selection:

\[ z_p = 1 \text{ if travel cost for pair } p \text{ is less through public network}; \]
\[ z_p = 0, \text{ otherwise}. \]

- Flow routing selection:

\[ w_{ij}^{pl} = 1 \text{ if the flow of pair } p \text{ traverses edge } ij \text{ of line } l; \]
\[ w_{ij}^{pl} = 0, \text{ otherwise}. \]

- Transfer variables:

\[ v_i^{pl} = 1 \text{ if flow of pair } p \text{ transfers to line } l \text{ at station } i; \]
\[ v_i^{pl} = 0, \text{ otherwise}. \]
Synthetizing the model

$$\text{RTND}_T: \quad \max \sum_{p \in P} g_p z_p$$

subject to

Length constraints
Alignment Location constraints
Routing Demand constraints
Location-Allocation constraints
Mode Splitting Demand constraints
Transfer constraints
Formulating the problem (i)

• **Objective function**: Trip coverage \( \max \sum_{p \in P} g_p z_p \)

• **Length constraints (LC)**:

\[ [1] \quad \sum_{ij \in E} d_{ij} x_{ij}^l \in \left[ length_{\text{min}}^l , length_{\text{max}}^l \right] ; \quad l \in L \]

\[ [2] \quad \sum_{l \in L} \sum_{ij \in E} d_{ij} x_{ij}^l \in \left[ Tlength_{\text{min}} , Tlength_{\text{max}} \right] \]
Formulating the problem (ii)

• Alignment location constraints (ALC)

[3] \[ \sum_{j \in N(O_i)} x_{O_i j}^l = 1; \quad l \in L \]

[4] \[ \sum_{i \in N(D_l)} x_{iD_l}^l = 1; \quad l \in L \]

[5] \[ y_{O_l}^l = y_{D_l}^l = 1; \quad l \in L \]

[6] \[ \sum_{j \in N(i)} x_{ij}^l = 2y_i^l; \quad i \in N \setminus \{O_i, D_l\}, l \in L \]

[7] \[ x_{ij}^l = x_{ji}^l; \quad ij \in E, \quad l \in L \]
Formulating the problem (iii)

- **Routing demand constraints (RDC):**

  \[ [8] \quad \sum_{j \in N(q)} u^p_{qj} = 1; \quad p = (q,r) \in P \]

  \[ [9] \quad \sum_{i \in N(q)} u^p_{iq} = 0; \quad p = (q,r) \in P \]

  \[ [10] \quad \sum_{i \in N(r)} u^p_{ir} = 1; \quad p = (q,r) \in P \]

  \[ [11] \quad \sum_{j \in N(r)} u^p_{rj} = 0; \quad p = (q,r) \in P \]

  \[ [12] \quad \sum_{i \in N(j)} u^p_{ij} - \sum_{k \in N(j)} u^p_{jk} = 0; \quad p = (q,r) \in P, \quad j \in N \setminus \{q,r\} \]
Formulating the problem (iv)

- Location-Allocation constraints (LAC):

\[
[13] \quad u^p_{ij} + z^p - 1 \leq \sum_{l \in L} x^l_{ij} ; \quad p \in P, \ ij \in E
\]

- Mode splitting demand constraints (MSDC)

\[
[14] \quad uc^PUB_p - uc^{PRIV}_p - M(1 - z^p) \leq 0; \quad p \in P
\]

where \( uc^PUB_p = \frac{1}{\lambda} \sum_{ij \in E} d_{ij} u^p_{ij} + \sum_{i \in N \setminus \{r\}, l \in L_i} \left( uc_i + \frac{1}{2f_l} \right) v^p_{il} ; \quad p \in P \)

and \( uc^{PRIV}_p \) are given.
Formulating the problem (v)

• Transfer constraints (TC):

[15] \( w_{ij}^{pl} \leq x_{ij}^l \); \( p \in P, \ ij \in E, \ l \in L \)

[16] \( u_{ij}^p + z_p - 1 \leq \sum_{l \in L} w_{ij}^{pl} \); \( p \in P, \ ij \in E \)

[17] \( u_{ij}^p - z_p + 1 \leq \sum_{l \in L} w_{ij}^{pl} \); \( p \in P, \ ij \in E \)

[18] \( \sum_{ij \in E, l \in L} w_{ij}^{pl} \leq M z_p \); \( p \in P, \ l \in L \)

[19] \( \sum_{j \in N(i)} w_{ij}^{pl} - \sum_{j \in N(i)} w_{ji}^{pl} \geq 2v_i^{pl} - 1 \); \( i \in N \setminus \{r\}, \ p \in P, \ l \in L \)

[20] \( \sum_{j \in N(i)} w_{ij}^{pl} - \sum_{j \in N(i)} w_{ji}^{pl} \geq 2v_i^{pl} \); \( i \in N \setminus \{r\}, \ p \in P, \ l \in L \)

\( x_{ij}^l, y_i^l, u_{ij}^p, z_p, w_{ij}^{pl}, v_i^{pl} \in \{0,1\} \).
About the complexity of real settings

RTND_T Size:

\[ |N| = 15, \quad |L| = 4, \quad |P| = 210, \quad |E| = 200, \quad |W| = 200, \]

<table>
<thead>
<tr>
<th>Binary Variables</th>
<th>(\chi_{ij}^l)</th>
<th>(y_i^l)</th>
<th>(u_{ij}^p)</th>
<th>(w_{ij}^{pl})</th>
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<th>RDC</th>
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Computational experiments (i)

- The integer model was implemented in GAMS, which calls CPLEX, on the network $G = (N, E)$.

Each node $i$ has an associated building cost $c_i$ and each edge $ij$ has associated a pair $(c_{ij}, d_{ij})$ of weights (building cost and the generalised cost of using the public edge).
Computational experiments (ii)

The origin-destination demands and the private cost for each demand pair are given in the following matrices:

\[
OD = \begin{bmatrix}
- & 9 & 26 & 19 & 13 & 12 \\
11 & - & 14 & 26 & 7 & 18 \\
30 & 19 & - & 30 & 24 & 8 \\
21 & 9 & 11 & - & 22 & 16 \\
14 & 14 & 8 & 9 & - & 20 \\
26 & 1 & 22 & 24 & 13 & - \\
\end{bmatrix}, \quad u_{C_{PRIV}} = \begin{bmatrix}
- & 1.6 & 0.8 & 2 & 2.6 & 2.5 \\
2 & - & 0.9 & 1.2 & 1.5 & 2.5 \\
1.5 & 1.4 & - & 1.3 & 0.9 & 2 \\
1.9 & 2 & 1.9 & - & 1.8 & 2 \\
3 & 1.5 & 2 & 2 & - & 1.5 \\
2.1 & 2.7 & 2.2 & 1 & 1.5 & - \\
\end{bmatrix}
\]
All transfer coefficients coincide

All transfer costs are equal to 0.75

<table>
<thead>
<tr>
<th>Length Range</th>
<th>Rapid Transit Network</th>
<th>Objective Function</th>
<th>Execution Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.5, 2.5]</td>
<td>[0.5, 2.5]</td>
<td>444 2 LINES</td>
<td>57.7 s</td>
</tr>
<tr>
<td>[0.5, 2.5]</td>
<td>[0.5, 2.5]</td>
<td>404 3 LINES</td>
<td>1666.72 s</td>
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</table>
# Low dispersion of transfer coefficients (2-L)

In the first line $\rightarrow$ 0.5; In the second $\rightarrow$ 1

<table>
<thead>
<tr>
<th>Length Range</th>
<th>Rapid Transit Network</th>
<th>Objective Function</th>
<th>Execution Time</th>
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<td>[0.5, 2.5]</td>
<td>LOW DISPERSION OF Transfer coefficients 2 LINES</td>
<td>456 2 LINES</td>
<td>95.23 s</td>
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<td>[0.5, 2.5]</td>
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<tr>
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<tr>
<td>[0.5, 3.5]</td>
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<td>470 2 LINES</td>
<td>61.2 s</td>
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<td><img src="Diagram_5.png" alt="Diagram 5" /></td>
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</table>
Low dispersion of transfer coefficients (3-L)

In the first line $\rightarrow 0.5$; In the second $\rightarrow 0.75$; In the third $\rightarrow 1.00$

<table>
<thead>
<tr>
<th>Length Range</th>
<th>Rapid Transit Network</th>
<th>Objective Function</th>
<th>Execution Time</th>
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<tbody>
<tr>
<td>[0.5, 1.5]</td>
<td>LOW DISPERSION OF Transfer coefficients</td>
<td>425 3 LINES</td>
<td>1252.69 s</td>
</tr>
<tr>
<td>[0.5, 1.5]</td>
<td>[0.5, 1.5]</td>
<td>1252.69 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 1.5]</td>
<td>[0.5, 1.5]</td>
<td>1252.69 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 2]</td>
<td>470 3 LINES</td>
<td>248.61 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 2]</td>
<td>[0.5, 2]</td>
<td>248.61 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 2]</td>
<td>[0.5, 2]</td>
<td>248.61 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 2.5]</td>
<td>470 3 LINES</td>
<td>248.61 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 2.5]</td>
<td>[0.5, 2.5]</td>
<td>248.61 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 2.5]</td>
<td>[0.5, 2.5]</td>
<td>248.61 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 3]</td>
<td>470 3 LINES</td>
<td>173.64 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 3]</td>
<td>[0.5, 3]</td>
<td>173.64 s</td>
<td></td>
</tr>
<tr>
<td>[0.5, 3]</td>
<td>[0.5, 3]</td>
<td>173.64 s</td>
<td></td>
</tr>
</tbody>
</table>
High dispersion of transfer coefficients (2-L)

In the first line $\rightarrow 0.25$; In the second $\rightarrow 1.25$

<table>
<thead>
<tr>
<th>Length Range</th>
<th>Rapid Transit Network</th>
<th>Objective Function</th>
<th>Execution Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.5, 2.5]</td>
<td>[0.5, 2.5]</td>
<td>447 2 LINES</td>
<td>64.63 s</td>
</tr>
<tr>
<td>[0.5, 3]</td>
<td>[0.5, 2.5]</td>
<td>456 2 LINES</td>
<td>46.81 s</td>
</tr>
<tr>
<td>[0.5, 3.5]</td>
<td>[0.5, 2.5]</td>
<td>456 2 LINES</td>
<td>38.81 s</td>
</tr>
</tbody>
</table>
**High dispersion of transfer coefficients (3-L)**

In the first line $\rightarrow 0.25$; In the second $\rightarrow 0.75$; In the third $\rightarrow 1.25$

<table>
<thead>
<tr>
<th>Length Range</th>
<th>Rapid Transit Network</th>
<th>Objective Function</th>
<th>Execution Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.5, 1.5]</td>
<td>[0.5, 1.5]</td>
<td>446 3 LINES</td>
<td>243.13 s</td>
</tr>
<tr>
<td>[0.5, 1.5]</td>
<td>[0.5, 1.5]</td>
<td>456 3 LINES</td>
<td>432.17 s</td>
</tr>
<tr>
<td>[0.5, 1.5]</td>
<td>[0.5, 1.5]</td>
<td>470 3 LINES</td>
<td>194.06 s</td>
</tr>
<tr>
<td>[0.5, 1.5]</td>
<td>[0.5, 1.5]</td>
<td>470 3 LINES</td>
<td>142.63 s</td>
</tr>
</tbody>
</table>
Low frequency of the train flow (2-L)

In the first line (the longest) … 12 trains per hour
In the second line ……………. 6 trains per hour
Average speed ……………….. 20 Km/h (for all lines)

<table>
<thead>
<tr>
<th>Congestion Level</th>
<th>Rapid Transit Network</th>
<th>Objective Function</th>
<th>Execution Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>LOW FREQUENCY OF TRAIN FLOW 2 LINES</td>
<td>331 2 LINES</td>
<td>494.47 s</td>
</tr>
<tr>
<td>1.5</td>
<td>444 2 LINES</td>
<td>97.76 s</td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>496 2 LINES</td>
<td>22.99 s</td>
<td></td>
</tr>
</tbody>
</table>
High frequency of the train flow (2-L)

In the first line (the longest) … 20 trains per hour
In the second line ……………. 10 trains per hour
Average speed ……………….. 20 Km/h (for all lines)

<table>
<thead>
<tr>
<th>Congestion Level</th>
<th>Rapid Transit Network</th>
<th>Objective Function</th>
<th>Execution Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>![Diagram 1.2]</td>
<td>414 2 LINES</td>
<td>253.14 s</td>
</tr>
<tr>
<td>1.5</td>
<td>![Diagram 1.5]</td>
<td>470 2 LINES</td>
<td>297.95 s</td>
</tr>
<tr>
<td>2.2</td>
<td>![Diagram 2.2]</td>
<td>496 2 LINES</td>
<td>0.63 s</td>
</tr>
</tbody>
</table>
General Conclusions

1. The best values of the objective function arise when the range for lines is wide.

2. When the dispersion of transfer costs increases (i.e., waiting time is heterogeneous) the flow shift is higher:
   - Best results for 2-line networks are obtained when the dispersion is low
   - Best results for 3-line networks are obtained when the dispersion is high
   - This fact remains although varies the range of line lengths.

3. The execution time descends when the dispersion of transfer cost and the range for the line lengths increase.

4. The train frequency along lines (function of the number of stations and the length of the lines) have a direct influence on the optimum design of the network.

5. The inclusion of transfers, usually considered only in those models formulated in tactical and operative levels, provides robustness to the networks designed by using our strategic model.
3. RESCHEDULING RAILWAY TIMETABLES IN PRESENCE OF PASSENGER TRANSFERS BETWEEN LINES WITHIN A TRANSPORTATION NETWORK

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University of Seville (SPAIN)

EWGT2013 – 16th Meeting of the EURO Working Group on Transportation
1. INTRODUCTION
2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES
3. PATTERN OF DEMAND BEHAVIOR
4. FORMULATION OF THE MODEL WITHOUT TRANSFERS
5. AN EXAMPLE
6. EXTENDING THE MODEL IN PRESENCE OF TRANSFERS
7. CONCLUSIONS
Given a railway infrastructure provided with different sections along a single transit line, the **Train Timetabling Problem** (TTP) consists of computing timetables that satisfy existing constraints and that optimize a single/multicriteria objective function for trains of both, passengers and/or cargo.

Timetable design is a **central problem** in railway planning with many interfaces with other classical problems: line planning, vehicle scheduling, and delay management.

The single-line Train Timetabling Problem is devoted to obtaining and optimizing timetables for trains with different levels of priority that share a railway line with single and multiple track sections.
1. INTRODUCTION (ii)

The **requirement for periodicity** of the timetables leads to the classification of TTP into Periodic (or cyclic) Train Timetabling and, on the other hand, Non-Periodic Train Timetabling.
1. INTRODUCTION (iii)

In Periodic Timetabling, the timetable is easy to remember for the passengers although its solutions can become inefficient when planning resources such as crews and rolling stock.


ARRIVAL project (http://arrival.cti.gr/, 2009).
1. INTRODUCTION (iv)

Non-Periodic Train Timetabling is relevant in presence of disturbances that can affect to the operativeness of train transit. The non-periodic train timetabling problem has been considered by most authors:


2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES (i)

Managers usually use running maps as graphic tools to plan train timetables. A running map is a time-space diagram where possible crossings of trains can be observed. Figure 1 shows the C4 line that belongs to the Madrid commuter railway network.

Figure 2 shows twenty-five instances of train schedules along the previous transit corridor.

Fig 1.: Line C4 (Parla-Atocha).  
Fig 2.: Train schedule
2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES (ii)

A transit line of high traffic density will generate in a labyrinthine tangle of polygonal lines, each of which will correspond to the hours of operation of a train, making infeasible a non-automated assessment of the possible alternatives.
2. GEOMETRIC REPRESENTATION OF TRAIN TIMETABLES (iii)


Context: SINGLE RAILWAYS CORRIDOR

H1. All trains run by the same railways corridor in one direction and at constant commercial speed along the way.

H2. There is a common time period \((h)\) that is used as unit for quantifying the time required for all service tasks sequenced.

H3. Time taken to travel without stopping between two consecutive stations is a multiple of \(h\).

H4. Minimum time required for boarding and alighting passengers on/from train is also \(h\).

H5. Temporary security margin between each pair of consecutive trains is a multiple of \(h\).

The above assumptions can be relaxed without altering the validity of the model.
Event-activity maps at stations along the corridor.
A uniform grid of squares of length $h$ (in terms of time) establishes feasible times for locating train maneuvers at each station of the line. Each active point in the event-activity map will indicate, simultaneously, arrival time (X-coordinate) and departure time (Y-coordinate) of a specific train.

Three trains passing through the station $k$
The sequence of stations (with stopping or not) along the railway line will correspond to a succession of temporary diagrams with active points indicating arrival-departure timetables.

Each timetable-point in the k-th diagram of activity will match to some other feasible point of the vertical segment that starts from its projection on the diagonal in the (k +1)-th activity-map.
3. PATTERN OF DEMAND BEHAVIOR (i)

Assume that arrival / departure times of trains at stations were previously set and are known by users.

Figure explains in percentage terms the travelers’ accumulation on the platform of station \( k \), due to the imminent arrival of the scheduled train \( i \) at time \( t_i \).

Time interval associated with the arrival of travelers to the platform is \( [t_i^-, t_i^+] \)

If the train \( i \) arrived on time, the whole population placed on platform could be transported, as shows the figure.

Usual demand behavior in terms of percentage of user’s presence at platform
Nevertheless, if train $i$ were delayed, the reaction of users when they know the existence of such delay would consist of initially waiting along a short certain period of time. Subsequently, the curve that models the percentage of population waiting would appear stabilized. After this period, the traveler population gradually decreases until disappearing.

If the train arrived late, only a portion of the population that normally waits could be transported.
Finally, if the train arrived and departed in advance, only users who were already placed on the platform could take the train. The other passengers will be coming in the usual way, because they were unaware of this schedule change (see figure).

Demand behavior if train departed in advance.

The option to wait a certain interval of time leads to the possibility of taking the next train.
3. PATTERN OF DEMAND BEHAVIOR (iv)

Assuming this behavior pattern, a **new time for the train arrival/departure** at the station can be determined, taking advantage of these overlapping demand curves.

The subsequent rescheduling of train timetables will have **the objective of minimizing the loss of passengers**.

**Two scenarios** can be considered depending on that passengers require transfers toward / from other network lines are (or not) at particular times.
4. FORMULATION OF THE MODEL WITHOUT TRANSFERS (i)

Indices and Sets

\( \begin{align*}
&i \in I \\
&k \in K \\
&u, v \in T \\
&(u, v) \in M_k
\end{align*} \)

- index identifying trains of set \( I \)
- index identifying cantons (or stations) of set \( K \)
- indices identifying the discretized time horizon \( T \)
- coordinates corresponding to temporary map \( M \) at station \( k \)

Parameters

\( \begin{align*}
&\alpha_{ik}^v \\
\end{align*} \)

population available to boarding to train \( i \) at station \( k \) and at time \( v \)

Variables

\( \begin{align*}
&x_{uv}^{ik} \\
\end{align*} \)

binary variable equals to 1 if train \( i \) is located at point \( (u, v) \) at station \( k \);

\[ x_{uv}^{ik} = 0, \text{ otherwise} \]

FORMULATION

\[ \text{(1) } \quad \text{Max} \quad \sum_{i \in I} \sum_{k \in K} \sum_{(u,v) \in M_k} \alpha_{ik}^v x_{uv}^{ik} \]

The objective function maximizes customers’ mobility by using the train system.
4. FORMULATION OF THE MODEL WITHOUT TRANSFERS (ii)

(1) The number of train schedules to be located must be exactly $|I|$.

(2) Forced passage through each station (with or without stopping) for all trains to be determined.

(3, 4) There can be no train arriving/departing from the k-th station if there was just another train operating.

(5) If there is a timetable-point located at position $(u, v)$ of the temporary map for the k-th station, then there must be another timetable point, at the $(k + 1)$-th station, on the $v$-th column.

(6) Limitation of the number of trains that can operate, according to the existing number of tracks.

(7) Binary nature of the decision variables.

$$
\sum_{i \in I} \sum_{(u,v) \in M_k} x_{uv}^{ik} = |I|; \quad k \in K
$$

$$
\sum_{k \in K} \sum_{(u,v) \in M_k} x_{uv}^{ik} = |K|; \quad i \in I
$$

$$
\sum_{i \in I} \sum_{(u,v) \in M_k} x_{uv}^{ik} \leq 1; \quad \sum_{i \in I} \sum_{(u,v) \in M_k} x_{uv}^{ik} \leq 1; \quad (u,v) \in M_k, \quad k \in K
$$

$$
x_{uv}^{ik} \leq \sum_{v' > v} x_{v'v'}^{i-1}; \quad (u,v) \in M_k, \quad i \in I, \quad k \in K (k \neq |K|)
$$

$$
\sum_{i \in I} \sum_{(u',v') \in M_k} x_{u'v'}^{ik} \leq n_k - x_{uv}^{ik}; \quad (u,v) \in M_k, \quad k \in K
$$

$$
x_{uv}^{ik} \in \{0, 1\}; \quad (u,v) \in T, \quad i \in I, \quad k \in K
$$
4. GREEDY ALGORITHM for selecting the $|I|$ better solutions in sequence

[Step 1] Set the mesh density of parameter $h$. Generate the sequence of temporary maps corresponding to the sections of railway line. Locate the existing timetable-points $(u,v)$.

[Step 2] Estimate populations for the remaining unmeasured timetable-points, according to the previous procedure, and obtain the maximum value $a_{uv}^\text{max}$.

[Step 3] Build an initial feasible graph $G_1$, composed of a sequence of maps ranging from the map $k = 1$ to $k = |K| - 1$ and whose edges connect points $(u, v)$ of the map $k$-th with points of the $(k + 1)$-th map, according to feasibility criteria.

[Step 4] i=1

While $i$ is less than $|I|$:

- Using a shortest path algorithm, determine the $i$-th optimal path connecting the two terminal stations of the line through the sequence of maps that represents graph $G_i$.
- Remove the feasible arcs used in the $i$-th path and those infeasible (isolated) arcs arising from the previous reduction. The new graph is denoted by $G_{i+1}$.
5. AN EXAMPLE (i)

- Assume a railway line that consists of 7 equi-spaced stations, separated from each other by a distance (travel time) equal to $h$. There are 3 vehicles crossing the line.
- Operating time [8:20 to 9:30] is discrete with periodicity of size $h = 2$ minutes.
- The arrival/departure timetables at stations are known by users (Table 1), along with the number of passengers at each station (Table 2).

<table>
<thead>
<tr>
<th>Station Number</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train 1: Arrival/Departure</td>
<td>8:26/8:28</td>
<td>8:30/8:32</td>
<td>8:34/8:36</td>
<td>8:38/8:42</td>
<td>8:44/8:48</td>
<td>8:50/8:52</td>
</tr>
<tr>
<td>Train 2: Arrival/Departure</td>
<td>8:38/8:40</td>
<td>8:42/8:44</td>
<td>8:46/8:48</td>
<td>8:50/8:54</td>
<td>8:56/9:00</td>
<td>9:02/9:04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Station Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train 1: Passengers</td>
<td>1417</td>
<td>1153</td>
<td>664</td>
<td>281</td>
<td>77</td>
<td>39</td>
<td>0</td>
</tr>
<tr>
<td>Train 2: Passengers</td>
<td>1143</td>
<td>756</td>
<td>359</td>
<td>113</td>
<td>23</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Train 3: Passengers</td>
<td>2131</td>
<td>1204</td>
<td>488</td>
<td>117</td>
<td>18</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>
5. AN EXAMPLE (ii)

Assume that, as consequence of an incident, the system operator must reduce the fleet size by one unit. Rescheduling train timetables must minimize the loss of users, by introducing advances or delays in the original schedules of the two vehicles which will remain operative. According to the previous model, the following distribution of passengers that access to stations is shown in table 1.

If train were not punctual, population waiting for boarding could be deterministically estimated (see table 2). Since it is assumed that the user loss for railway system is only caused by decisions of putting advanced or delayed schedules, the sequence of blue cells indicates optimal reprogramming of the two feasible schedules.
The solution after applying a myopic methodology (cancel the train that serves the smallest number of users) can be compared with that obtained by applying the model (by introducing small advances or delays in the starting times to reduce the loss of users).

The results obtained are summarized in Table 4.

<table>
<thead>
<tr>
<th>TABLE 4: PASSENGERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL STATUS</td>
</tr>
<tr>
<td>TRAIN 1</td>
</tr>
<tr>
<td>TRAIN 2</td>
</tr>
<tr>
<td>TRAIN 3</td>
</tr>
<tr>
<td>TOTAL</td>
</tr>
<tr>
<td>% LOSS</td>
</tr>
</tbody>
</table>
If transit corridor intersects with others transit lines at specific stations, as is shown in figure, the determination of new timetables should ensure the transfer of passengers between trains from different line runs at such interchange stations.
Two strategies can be considered:

- Imposing synchronization between the timetables of these lines; that is, a solution can be accepted only if the connection between them is feasible (Scenario 2.1).

- Rewarding the possibility of providing transfers for passengers of external lines towards concurrent expeditions of the internal line by means of a weighting factor (Scenario 2.2).

For instance, Figure shows the timetable-point (filled in red) of another line (line A) when arrives/departs at/from station 4 at times $u=4$ and $v=8$, respectively. If the synchronization between the timetables of these lines were imposed, the feasible subset of timetable-points, where transfer is preserved, would coincide with the set of unfilled points in magenta color.
Consistently with the notation used for decision variables in the model, let be binary input data which is equal to 1 if train $j$ (of an external line whose arrival/depart timetables are given) is located at timetable-point $(u, v)$ at station $s$; otherwise, its value would be 0.

$$y_{uv}^{js} \quad \text{for all } j \in J, (u, v) \in M_s, s \in S$$
6. EXTENDING THE MODEL IN PRESENCE OF TRANSFERS (iv)

NEW Indices and Sets

- \( j \in J \) index that identifies trains of other transit lines concurrent with lines runs of set \( J \)
- \( s \in S \subset K \) index that enumerates the subset of stations that allow transfers to the travelers.
- \( F_s(\text{traveler}) \subset M_s \) subset of timetable-points in the temporary map \( M \) of station \( s \) where transfers between two transit lines can be carried out.

FORMULATING the MODEL FOR SCENARIO 2.1

Objective (1), constraints (2)-(7), and additionally:

\[
(8) \quad y_{uv}^{js} \leq \sum_{i \in I} \sum_{(u',v') \in F_s(u,v)} x_{u'v'}^{is} \quad j \in J, (u,v) \in M_s, s \in S
\]

Constraints (8) establish that if there is an active (i.e., \( y_{uv}^{js} = 1 \)) timetable-point located at position \((u, v)\) of the temporary map for the \( s\)-th station of an outside line \( j \), then there must be at least another active timetable-point at the same station for synchronizing transfers from/toward line runs \( i \) of the inner transit line \( I \).
EXTENDING THE MODEL IN PRESENCE OF TRANSFERS (v)

For this context, it is necessary to distinguish between users who enter in the system from outside and passengers who previously entered into the system with the certainty of being able to make a transfer to another line already. Objective to maximize must take into account this division of populations and asymmetrically favor one over the other population by using a weighting factor $\mu > 1$.

Let $b_{uv}^{jk}$ be a real input data which represents the population available to transferring from train $j$ at station $k$ and at timetable-point $(u, v)$.

FORMULATING the MODEL FOR SCENARIO 2.2

Redefining the objective (1’):

$\text{(1')} \quad \text{Max} \quad \sum_{i \in I} \sum_{k \in K} \sum_{(u,v) \in M_k} (a_{ik}^{uv} + \mu \sum_{j \in J} \sum_{(u',v') \in F_k(u,v)} b_{ujv'}^{jk}) x_{uv}^{ik}$

If $k$ is not an interchange station, then $F_k(u,v) = \emptyset$ and the second additive term is cancelled.

Therefore, objective (1’) and constraints (2)-(8) constitute a procedure for maximizing mobility of travelers who enter in the system after rescheduling, by ensuring the option of transferring from/towards other external lines at interchange stations.
7. CONCLUSIONS

A geometric approach to determine the redistribution of service along a rail corridor has been introduced.

Motivation for rescheduling railway timetables is caused by the forced reduction of fleet size due to accidents, strikes and other sources of train delays and cancellations.

Two scenarios have been modelled: a context without considering transfers from/towards other transit lines, and a setting where the existence of transfers between lines must be preserved although the service must be rescheduled.

A common approach for these scenarios has been developed by using a geometrical representation of train timetables at stations. The associated formulations are Integer Linear Programming models, where the number of decision variables can be reduced according to different constraints imposed by the structural and fleet capacities.

The theoretical development has been illustrated with a non-sophisticated example in order to clarify the used concepts.
4. A location-routing problem arising in the operations of a courier services provider: A case study from Fuerteventura Island in Spain

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\textit{ISOLDE XII}  
Outline

1 Introduction
2 Background
3 Formulation
4 Computational Results
5 Conclusions
The Canary Islands is a Spanish archipelago, located just off the northwest Atlantic coast of mainland Africa.
The Canary Islands comprise seven major islands: Lanzarote, Fuerteventura, Gran Canaria, Tenerife, La Palma, La Gomera and El Hierro.
Fuerteventura, with a surface of 1,660 km², is the second-most extensive island of the archipelago, as well as the second most oriental. It has been declared a Biosphere reserve by Unesco. The island of Fuerteventura has a population of 103,492 inhabitants (according to 2010 data) and administratively belongs to the province of Gran Canaria.
As the airport of Fuerteventura has only a domestic category, daily at 23:30 hours, a boat of Naviera Armas leaves the harbor of Las Palmas de Gran Canaria towards Puerto del Rosario, the Fuerteventura main town, carrying postal items to the island. The main post office, located in Puerto del Rosario, manages the reception of goods (boat arrives by 7:00 hours next day) and then starts a distribution route in order to delivery packages coming from the outside to the other Fuerteventura offices and, simultaneously, collect the items produced from these offices to other destinations, both inland and abroad.
The complex orography of the island of Fuerteventura and the geographic dispersion of post offices hinder completing the distribution route of abroad shipments, with enough time to reuse the same boat on their return journey (11:30 hours).
Currently, managers must use for this purpose another boat of the same company, that sails from the port of Morro Jable (18:00 hours) towards Las Palmas de Gran Canaria.
The existence of two main offices for connecting with abroad generates organizational problems and cost-overruns.
Managers of the postal service have empirically checked that any delivery tour which left and returned from/to Puerto del Rosario, would take more than 11 hours in visiting all the sites. Therefore, solving the problem of mail delivery can’t be to design a single delivery route.

A tour for visiting all the postal offices and returning to your starting point.
In practice, Puerto del Rosario is used as an enter gate (at 7:00 horas) while the port of Morro Jable plays the role of exit gate for the island, because the ship's departure is later (at 18:00 horas).

A dispatching network with an ending point different than the starting point.
The aim of this section is to formulate a model that determines an optimal distribution network, composed of a main cyclical route that starts and ends at a deposit point (Puerto del Rosario) and other provisioning sub-routes, such that a restricted time window can’t be violated.

Figure shows a feasible dispatching network, that takes less than 4 hours in the total distribution of mailing. Is it optimal?
**Fuerteventura Island:**

- 26 Municipalities
- 10 Crossroads (checkpoints)
- 48 edges
- 650 o-d pairs
- ... and around 5h to carry out the mail distribution
Definition. (Vehicle Routing Problem - VRP)
The VRP consists of designing a set of minimal cost routes that visit a set of nodes of a network. It is usually assumed that all routes start and end in a especial node called depot.

Example.
**Definition.** (Ring star)
A ring star in $G$ is a subgraph that can be decomposed into a cycle (or ring) and a set of vertices connected to the graph (also called assignments) using a single arc.

**Definition.** (Multi Ring Star Problem - mRSP)
The mRSP consists of designing a set of $m$ minimal cost ring stars in such a way that each node of $G$ is connected to a ring star.

**Example.**
Definition. (Vehicle Routing Problem with Pickup and Deliveries- VRPPD) A VRP with Pickups and Deliveries (VRPPD) assumes a set of transportation requests consisting of different commodities that have to be transported between origins and destinations of a network.

Example.
Optional precedences

**Definition.** *(Optional precedence)*
Given a transportation request \( i \) between an o-d node pair of a network, we say that the precedence of \( i \) is *optional* if the pickup node of the request can be visited after the delivery node in exchange of a cost.

**Example.**

![Diagram](image)

**Remark.** Assignments and optional precedences are required in the routes configuration of a courier service that is not able to visit all nodes in a network ensuring all precedences.
Let $G$ be a complete graph where $n$ different commodities want to be shipped between $n$ different pairs of nodes of the graph.

**Definition.** *(mRSPPD)*
The multi Ring Star Problem with Pickups and Deliveries (mRSPPD) consists of designing a set of minimal cost ring stars in such a way that each node shipping/receiving one/several commodities is connected to a ring star and the number of satisfied precedences is maximized.


Laporte, G. and Rodríguez-Martín, I. (2007) “Locating a cycle in a transportation or a telecommunications network”.


*Discrete Applied Mathematics*, vol. 35(0): 103–108.


VRPs and checkpoints


Scientific Report.


### Formulation: Sets and Parameters

- **$k \in K$**: set of vehicles
- **$n$**: number of pickup requests
- **$P \subset N$**: set of pickup requests ($P = \{1, \ldots, n\}$)
- **$D \subset N$**: set of delivery requests ($D = \{n+1, \ldots, 2n\}$)
- **$i \in I = P \cup D$**: set of transportation requests
- **$\{0, 2n+1\} \subset N$**: initial and final depot nodes
- **$U$**: set of origin-destination nodes
- **$u \in U_i \subseteq U$**: set of origin-destination nodes to which request $i$ can be allocated
- **$e \in E$**: set of arcs ($e = (e^-, e^+)$, $e^-, e^+ \in N$)
- **$E_i^+ \in E$**: set of arcs leaving node $i \in N$
- **$E_i^- \in E$**: set of arcs entering node $i \in N$
- **$d_u$**: service time at node $u$

### Costs

- **$c_e^X$**: routing cost incurred when edge $e$ is selected
- **$c_{iu}^Y$**: cost incurred when request $i$ is allocated to node $u$
- **$c_i^Z$**: cost incurred when precedence constraints of request $i$ are not satisfied
Formulation: Variables

\[ x_e^k \] binary variable equal to 1 if arc \( e \) is covered by vehicle \( k \)

\[ t_u^k \] positive variable representing departure time at node \( u \)

\[ z_i \] binary variable equal to 1 if the precedence of request \( i \) is not satisfied

\[ y_{iu} \] binary variable equal to 1 if request \( i \) is allocated to node \( u \in U_i \)
Formulation

\[
\text{Min } \lambda_1 \sum_{k \in K} \sum_{e \in E} c_e^X x_e^k + \lambda_2 \sum_{i \in I} \sum_{u \in U_i} c_i^Y y_{iu} + \lambda_3 \sum_{i \in I} c_i^Z z_i
\]  

\text{s.t.: } \sum_{e \in E_{0}^{+}} x_e^k = \sum_{e \in E_{2n+1}^{-}} x_e^k = 1 \quad \quad \quad k \in K \tag{1b}
\]

\[
\sum_{e \in E_u^{+}} x_e^k - \sum_{e \in E_u^{-}} x_e^k = 0 \quad \quad \quad u \in U, k \in K \tag{1c}
\]

\[
\sum_{u \in U_i} y_{iu} = 1 \quad \quad \quad i \in I \tag{1d}
\]

\[
y_{iu} \leq \sum_{k \in K} \sum_{e \in E_u^{+}} x_e^k \leq 1 \quad \quad \quad i \in I, u \in U_i \tag{1e}
\]

\[
(t_u^k + d_u + c_e) x_e^k \leq t_u'^k \quad \quad \quad e \in E, u = e^-, u' = e^+, k \in K \tag{1f}
\]

\[
(1 - z_i) \sum_{u \in U_i} t_u^k y_{iu} \leq \sum_{u \in U_{i+n}} t_u^k y_{i+n,u} \quad \quad \quad i \in I, k \in K \tag{1g}
\]

\[
x_e^k \in \{0, 1\} \quad \quad \quad e \in E, k \in K \tag{1h}
\]

\[
y_{iu} \in \{0, 1\} \quad \quad \quad i \in I, u \in U_i \tag{1i}
\]

\[
z_i \in \{0, 1\} \quad \quad \quad i \in I \tag{1j}
\]
Objective function minimizes the assignment costs

Min \( \lambda_1 \sum_{k \in K} \sum_{e \in E} c_e^X x_e^k + \lambda_2 \sum_{i \in I} \sum_{u \in U_i} c_i^Y y_{iu} + \lambda_3 \sum_{i \in I} c_i^Z z_i \)  

s.t.: 
\( \sum_{e \in E_0^+} x_e^k = \sum_{e \in E_{2n+1}} x_e^k = 1 \) \hspace{1cm} k \in K \hspace{1cm} (1b) 

\( \sum_{e \in E_u^+} x_e^k - \sum_{e \in E_u^-} x_e^k = 0 \) \hspace{1cm} u \in U, k \in K \hspace{1cm} (1c) 

\( \sum_{u \in U_i} y_{iu} = 1 \) \hspace{1cm} i \in I \hspace{1cm} (1d) 

\( y_{iu} \leq \sum_{k \in K} \sum_{e \in E_u^+} x_e^k \leq 1 \) \hspace{1cm} i \in I, u \in U_i \hspace{1cm} (1e) 

\( (t_u^k + d_u + c_e)x_e^k \leq t_u^k \) \hspace{1cm} e \in E, u = e^-, u' = e^+, k \in K \hspace{1cm} (1f) 

\( (1 - z_i) \sum_{u \in U_i} t_u^k y_{iu} \leq \sum_{u \in U_{i+n}} t_u^k y_{i+n,u} \) \hspace{1cm} i \in I, k \in K \hspace{1cm} (1g) 

\( x_e^k \in \{0, 1\} \) \hspace{1cm} e \in E, k \in K \hspace{1cm} (1h) 

\( y_{iu} \in \{0, 1\} \) \hspace{1cm} i \in I, u \in U_i \hspace{1cm} (1i) 

\( z_i \in \{0, 1\} \) \hspace{1cm} i \in I \hspace{1cm} (1j)
Constraints ensure that at least one vehicle departs and arrives from/to the depot.

\[
\begin{align*}
\text{s.t.:} & \quad \sum_{e \in E_0^+} x_e^k = \sum_{e \in E_{2n+1}^-} x_e^k = 1 & & k \in K \tag{1a} \\
& \sum_{e \in E_u^+} x_e^k - \sum_{e \in E_u^-} x_e^k = 0 & & u \in U, \ k \in K \tag{1b} \\
& \sum_{u \in U_i} y_{iu} = 1 & & i \in I \tag{1c} \\
& y_{iu} \leq \sum_{k \in K} \sum_{e \in E_u^+} x_e^k \leq 1 & & i \in I, \ u \in U_i \tag{1d} \\
& (t_u^k + d_u + c_e)x_e^k \leq t_u^k & & e \in E, \ u = e^-, \ u' = e^+, \ k \in K \tag{1e} \\
& (1 - z_i) \sum_{u \in U_i} t_u^k y_{iu} \leq \sum_{u \in U_{i+n}} t_u^k y_{i+n,u} & & i \in P, \ k \in K \tag{1f} \\
& x_e^k \in \{0, 1\} & & e \in E, \ k \in K \tag{1g} \\
& y_{iu} \in \{0, 1\} & & i \in I, \ u \in U_i \tag{1h} \\
& z_i \in \{0, 1\} & & i \in I \tag{1i} \\
\end{align*}
\]
Min $\lambda_1 \sum_{k \in K} \sum_{e \in E} c_e^X x_e^k + \lambda_2 \sum_{i \in P} \sum_{u \in U_i} c_i^Y y_{iu} + \lambda_3 \sum_{i \in P} c_i^Z z_i$

Constraints ensure flow conservation in the nodes of the network.

$\sum_{e \in E_{u}^+} x_e^k - \sum_{e \in E_{u}^-} x_e^k = 0$

$\sum_{u \in U_i} y_{iu} = 1$

$y_{iu} \leq \sum_{k \in K} \sum_{e \in E_{u}^+} x_e^k \leq 1$

$(t_u^k + d_u + c_e)x_e^k \leq t_u^k$

$(1 - z_i) \sum_{u \in U_i} t_u^k y_{iu} \leq \sum_{u \in U_{i+n}} t_u^k y_{i+n,u}$

$x_e^k \in \{0, 1\}$

$y_{iu} \in \{0, 1\}$

$z_i \in \{0, 1\}$

$k \in K$

$u \in U, k \in K$

$i \in I$

$i \in I, u \in U_i$

$e \in E, u = e^-, u' = e^+, k \in K$

$i \in P, k \in K$

$e \in E, k \in K$

$i \in I, u \in U_i$

$i \in I$
Min $\lambda_1 \sum_{k \in K} \sum_{e \in E} c_e x_e^k + \lambda_2 \sum_{i \in P} \sum_{u \in U_i} c_i y_{iu} + \lambda_3 \sum_{i \in P} c_i^T z_i$

$$\sum_{e \in E^+_u} x_e^k = \sum_{e \in E^-_u} x_e^k = 1 \quad k \in K$$

$$\sum_{u \in U_i} y_{iu} = 1 \quad i \in I$$

$$y_{iu} \leq \sum_{k \in K} \sum_{e \in E^+_u} x_e^k \leq 1 \quad i \in I, u \in U_i$$

$$(t^k_u + d_u + c_e)x_e^k \leq t^k_{u'} \quad e \in E, u = e^-, u' = e^+, k \in K$$

$$(1 - z_i) \sum_{u \in U_i} t^k_u y_{iu} \leq \sum_{u \in U_{i+n}} t^k_u y_{i+n,u} \quad i \in P, k \in K$$

$$x_e^k \in \{0, 1\} \quad e \in E, k \in K$$

$$y_{iu} \in \{0, 1\} \quad i \in I, u \in U_i$$

$$z_i \in \{0, 1\} \quad i \in I$$
Formulation

Min $\lambda_1 \sum_{k \in K} \sum_{e \in E} c^X_e x^k_e + \lambda_2 \sum_{i \in I} \sum_{u \in U_i} c^Y_i y_{iu} + \lambda_3 \sum_{i \in I} c^Z_i z_i$ \hspace{1cm} (1a)

Constraints ensure that the node allocated to a request is at most visited once.

$\sum_{u \in U_i} y_{iu}$ \quad (1b)

$y_{iu} \leq \sum_{k \in K} \sum_{e \in E_u^k} x^k_e \leq 1$ \quad (1c)

$(t_u^k + d_u + c_e)x^k_e \leq t_u^k$ \quad (1d)

$(1 - z_i) \sum_{u \in U_i} t^k_u y_{iu} \leq \sum_{u \in U_{i+n}} t^k_u y_{i+n,u}$ \quad (1e)

$x^k_e \in \{0, 1\}$ \hspace{1cm} (1f)

$y_{iu} \in \{0, 1\}$ \hspace{1cm} (1g)

$z_i \in \{0, 1\}$ \hspace{1cm} (1h)

$e \in E, u = e^-, u' = e^+, k \in K$ \hspace{1cm} (1i)

$k \in K$ \hspace{1cm} (1j)

$m \in M$ \hspace{1cm} (1k)

$m = |M|$ \hspace{1cm} (1l)

$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ \hspace{1cm} (1m)

$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ \hspace{1cm} (1n)
Formulation

\[
\text{Min } \lambda_1 \sum_{k \in K} \sum_{e \in E} c_e^X x_e^k + \lambda_2 \sum_{i \in I} \sum_{u \in U_i} c_i^Y y_{iu} + \lambda_3 \sum_{i \in I} c_i^Z z_i
\]

\[\text{s.t.:}\]
\[\sum_{e \in E_0^+} x_e^k = \sum_{e \in E_{2n+1}^+} x_e^k = 1 \quad \quad k \in K\]
\[\sum_{e \in E_u^+} x_e^k - \sum_{e \in E_u^-} x_e^k = 0 \quad \quad i \in I, \quad u \in U_i, \quad k \in K\]
\[\sum_{k \in K} x_e^k \leq 1 \quad \quad e \in E, \quad u = e^-, \quad u' = e^+, \quad k \in K\]
\[
(1 - z_i) \sum_{u \in U_i} t_{u}^k y_{iu} \leq \sum_{u \in U_{i+n}} t_{u}^k y_{i+n,u} \quad \quad i \in P, \quad k \in K\]

\[x_e^k \in \{0, 1\}, \quad y_{iu} \in \{0, 1\}, \quad z_i \in \{0, 1\}\]
Min $\lambda_1 \sum_{k \in K} \sum_{e \in E} c_e^X x_e^k + \lambda_2 \sum_{i \in I} \sum_{u \in U_i} c_i^Y y_{iu} + \lambda_3 \sum_{i \in I} c_i^Z z_i$ (1a)

s.t.: $\sum_{e \in E_0^+} x_e^k = \sum_{e \in E_{2n+1}^-} x_e^k = 1$ \quad $k \in K$ (1b)

$\sum_{e \in E_u^+} x_e^k - \sum_{e \in E_u^-} x_e^k = 0$ \quad $u \in U, k \in K$ (1c)

$\sum_{u \in U_i} y_{iu} = 1$ \quad $i \in I$ (1d)

$(t_u^k + d_u + c_e)x_{e}^{k - u'}$ \quad $e \in E, u = e^-, u' = e^+, k \in K$ (1f)

$(1 - z_i) \sum_{u \in U_i} t_u^k y_{iu} \leq \sum_{u \in U_{i+n}} t_u^k y_{i+n,u}$ \quad $i \in I, k \in K$ (1g)

$x_e^k \in \{0, 1\}$ \quad $e \in E, k \in K$ (1h)

$y_{iu} \in \{0, 1\}$ \quad $i \in I, u \in U_i$ (1i)

$z_i \in \{0, 1\}$ \quad $i \in I$ (1j)

**Constraints control those precedences that are satisfied.**
Side Constraints

Max route length constraint:
\[
\sum_{e \in E} c_e^X x_e^k \leq UB^X \quad k \in K
\]

Max allocation distance:
\[
c_a^Y y_a \leq UB^Y \quad k \in K
\]

Minimal preceding satisfaction:
\[
\sum_{i \in P} c_i^Z z_i \leq UB^Z \quad k \in K
\]

Specific preceding satisfaction:
\[
z_i = 0 \quad i \in P'
\]
**VRP solution (1 vehicle)**

**Data set:**
- 26 Municipalities
- 10 Checkpoints
- 48 edges
- 650 o-d pairs
VRP solution (2 vehicles)

Data set:
- 26 Municipalities
- 10 Checkpoints
- 48 edges
- 650 o-d pairs

A location-routing problem arising in the operations of a courier services provider
mRSP solution (1 vehicle)

Data set:
- 26 Municipalities
- 10 Checkpoints
- 48 edges
- 650 o-d pairs
mRSP solution (2 vehicles)

Data set:
- 26 Municipalities
- 10 Checkpoints
- 48 edges
- 650 o-d pairs
A 1V-RSPPD solution returning to Puerto del Rosario and shipping all mails from node 1 to each node.
The 2V-RSPPD solution returning to Puerto del Rosario and shipping all mails from node 1 to each node.
Conclusions and future research

1. We present a solution approach to face the courier distribution management of the Island of Fuerteventura (Spain).
2. A MIP formulation is proposed with several side constraints in order to fit the model according to the different requirements of the problem.
3. The set of solutions provided allow to complete all mail distribution according to different trade-offs.