Joyal theorems for homotopical species

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Joyal, in the early 1980s, introduced combinatorial species and the accompanying theory of analytic functors in order to explain the use of generating functions in combinatorics. He proved that an endofunctor of Set is analytic iff it preserves filtered colimits, cofiltered limits and weak pullbacks. Work in progress with David Gepner extends this theory from sets to infinity groupoids, introducing (various flavours of) homotopical species, supposed to be thought of as data structures holding homotopy cardinalities, i.e. 'counting' problems with values in homotopy types; as an example Grassmannians 'count' vector subspaces, and form the coefficients for a homotopical species in two variables. Baez-Dolan 'stuff types' (a categorification of the harmonic oscilator) is a special case of homotopical species. For each of the flavours of homotopical species we prove a 'Joyal theorem'. Assuming the theory of quasi-categories, the proofs are actually easier in the homotopical setting than in the classical case, essentially because the technically inconvenient notion of weak pullback evaporates, and the notion of 'analytic' merges with 'polynomial', allowing exploitation of the representability features of polynomial functors. For the simplest of the notions of homotopical species, the exactness conditions are: preservation of sifted colimits and connected limits. The classical Joyal theorem results as a corollary by taking connected components. In my talk I will first review the classical theory of species, then recall some notions from the theory of quasi-categories and outline the proof of our 'Joyal theorem' for homotopical species. If time permits I will speculate about future directions of the theory.