Enumeration of self-dual planar maps

Anna de Mier

Universitat Politècnica de Catalunya, Barcelona

Brief summary

1. ~~Enumeration of self-dual~~ planar maps
2. Enumeration of ~~self-dual~~ planar maps
3. Enumeration of self-dual planar maps

Planar maps

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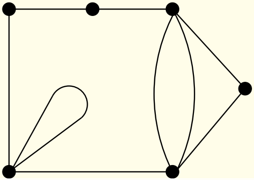

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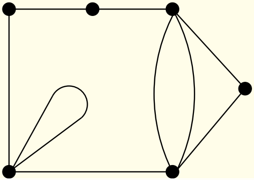

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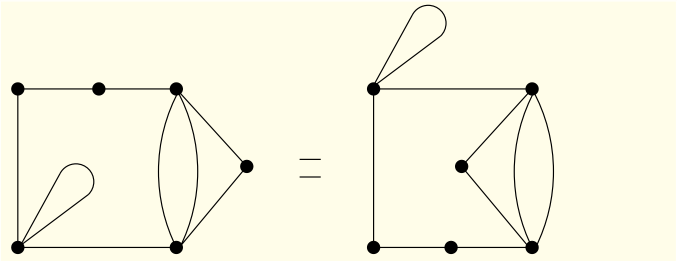

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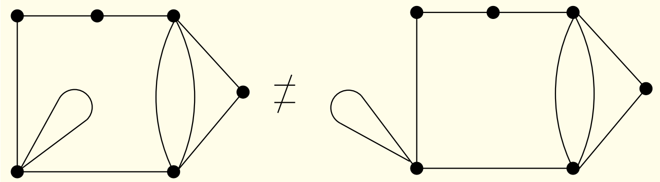

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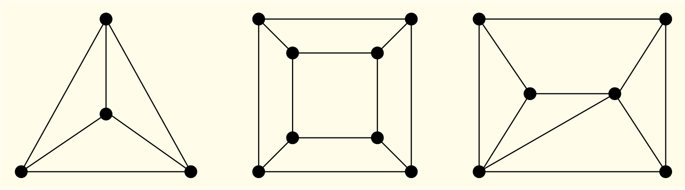

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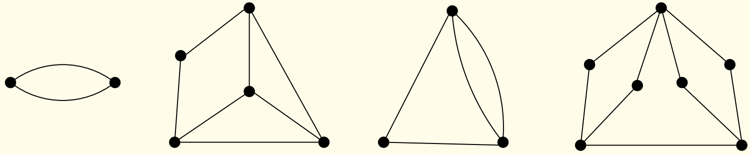

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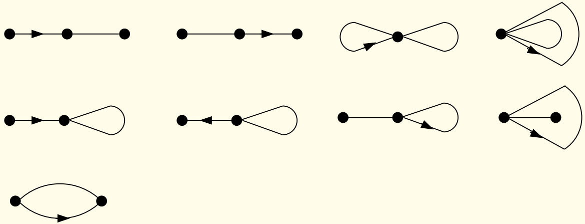
(When drawing on the plane, the root face is always the one to the right of the root edge)



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The generating function for 3-connected rooted maps is

$$z^6 + 4z^8 + 6z^9 + 24z^{10} + 66z^{11} + 214z^{12} + \cdots$$

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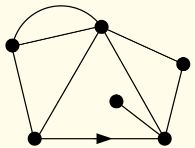
(The generating function of a sequence $a_0, a_1, a_2 \ldots$ is the series

$$A(z) = a_0 + a_1 z + a_2 z^2 + \cdots \qquad)$$

Topics in map enumeration

- Enumeration of maps according to other restrictions/parameters
- Enumeration of non-rooted maps
- Relation with other combinatorial objects and bijective proofs

The dual of a rooted map



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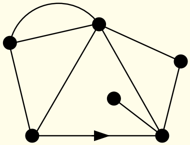

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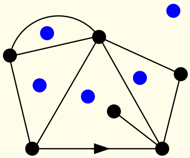

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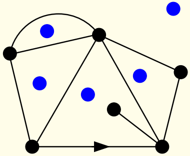

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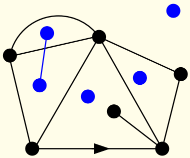

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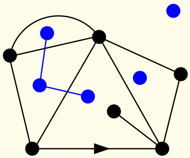

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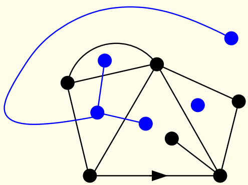

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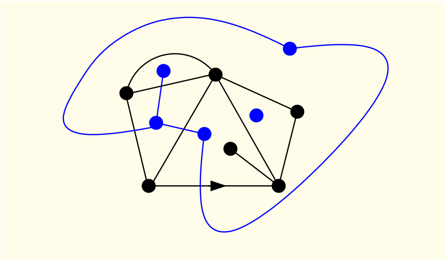

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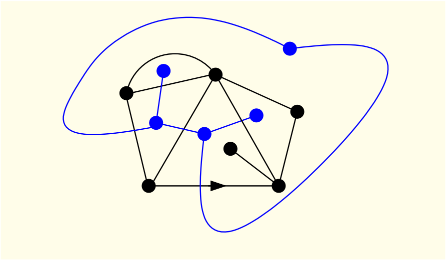

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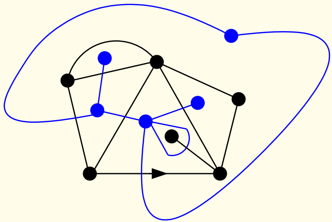

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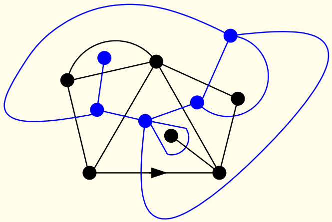

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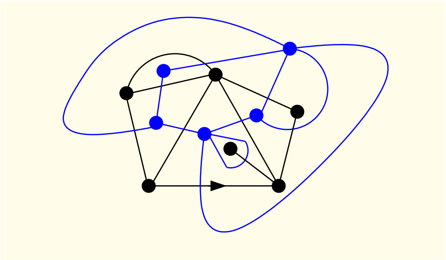

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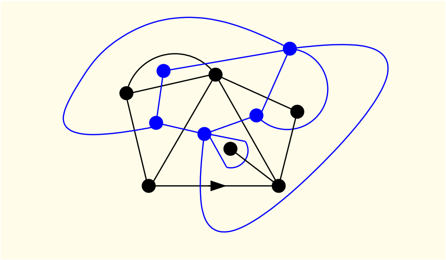

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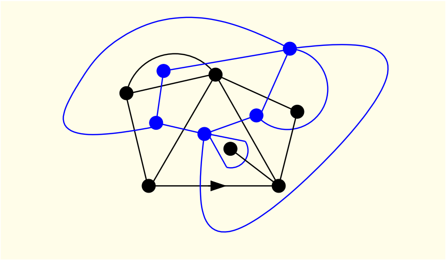

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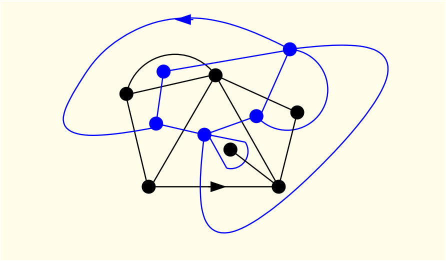

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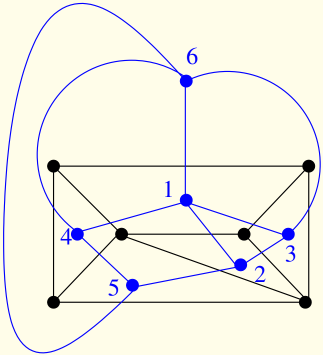

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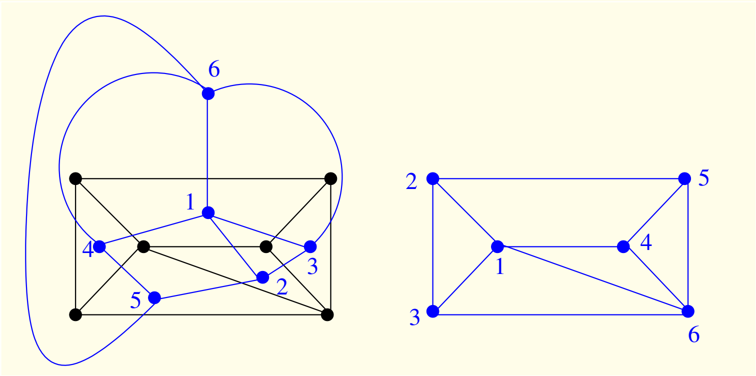

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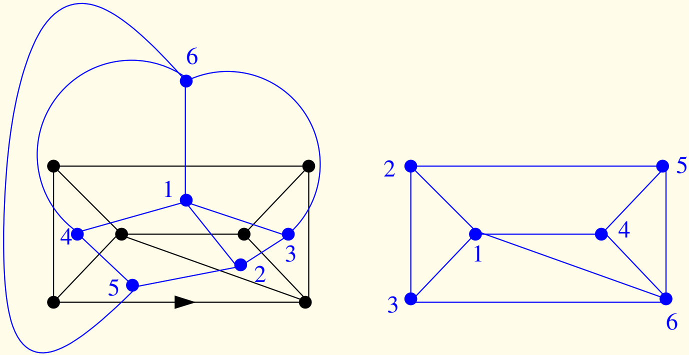

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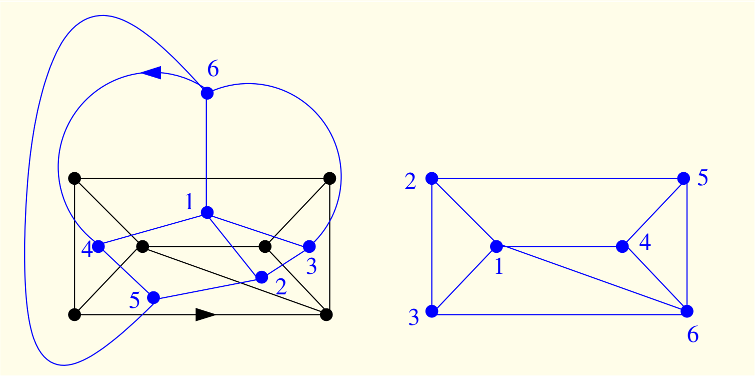

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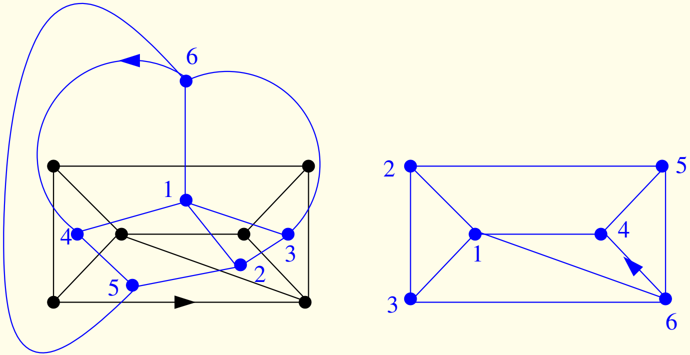

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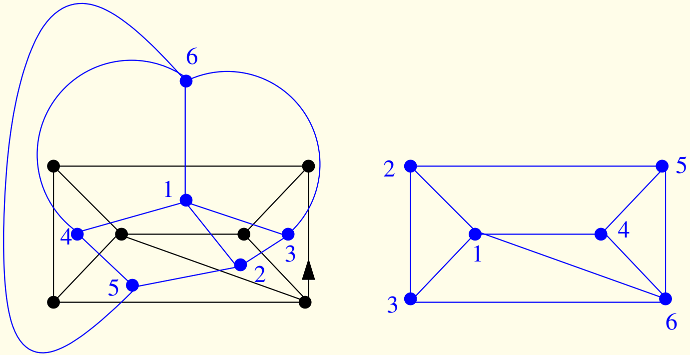

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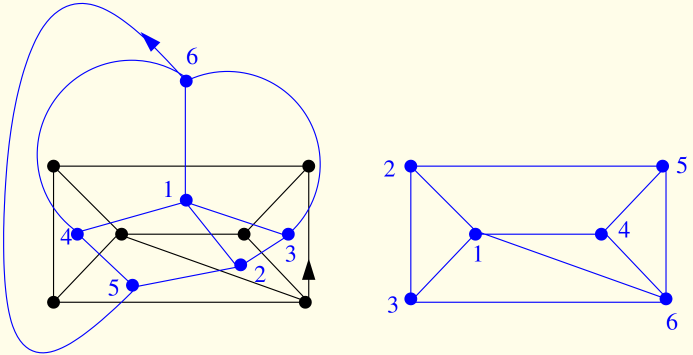

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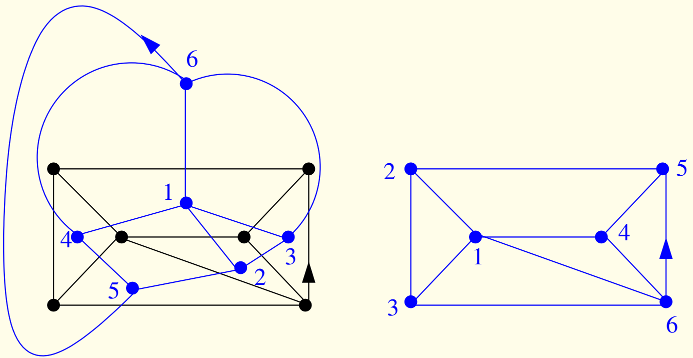

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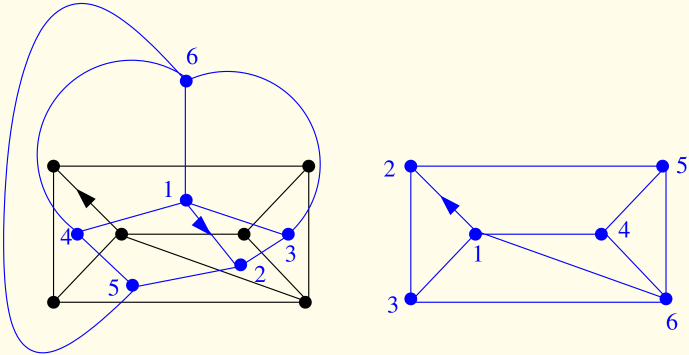


The numbers of self-dual rooted maps

Thm (Kitaev, de Mier, Noy 14)

- The number of self-dual maps with $2n$ edges is

$$\frac{3^n}{n+1}\binom{2n}{n}$$

- The number of 2-connected self-dual maps with $2n$ edges is

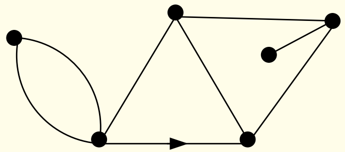
$$\frac{1}{n}\binom{3n-2}{n-1}$$

- The generating function for 3-connected self-dual maps is

$$\frac{1-2z-2z^2-\sqrt{1-4z}}{2(z+2)} = z^3 + 2z^4 + 6z^5 + 18z^6 + \cdots$$

The quadrangulation associated to a map

There is a well-known bijection between maps and loopless quadrangulations (i.e., maps where all faces have 4 sides)



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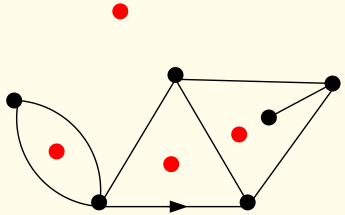

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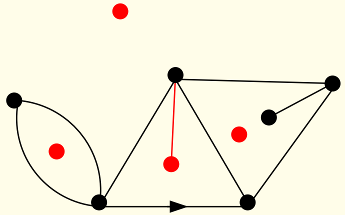

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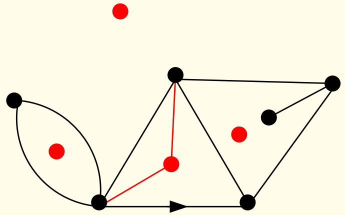

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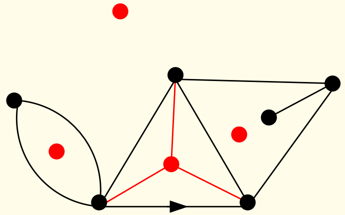

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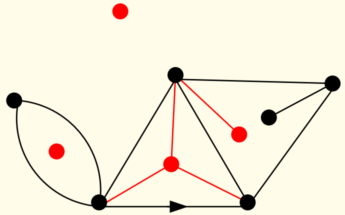

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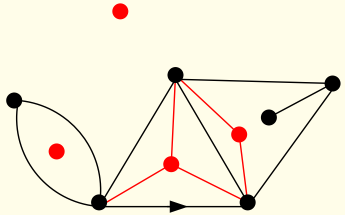

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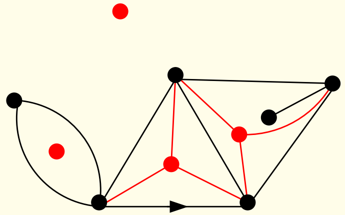

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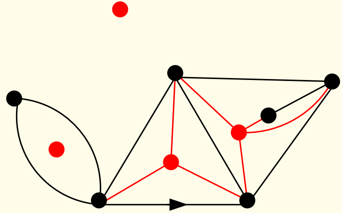

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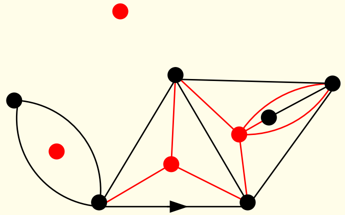

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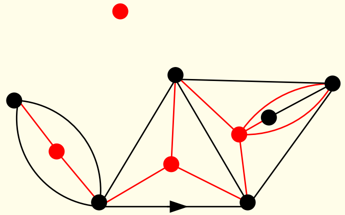

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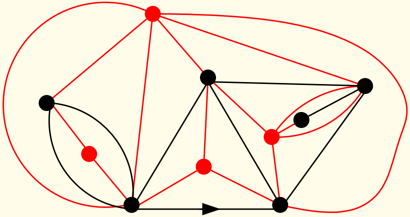

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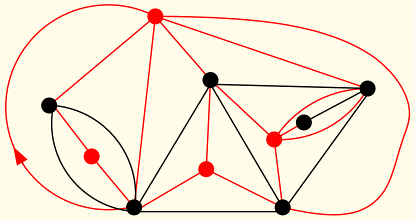

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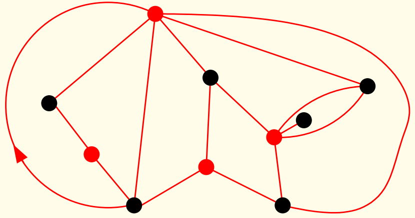

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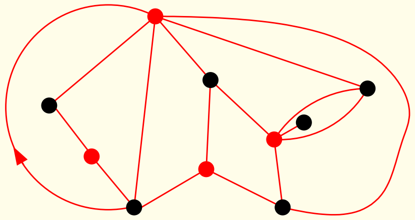
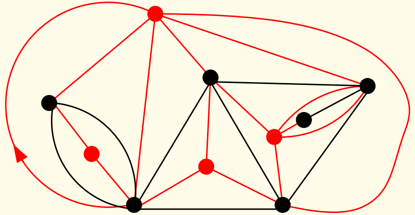


If the map has n edges, the quadrangulation has n faces

The quadrangulation associated to a map

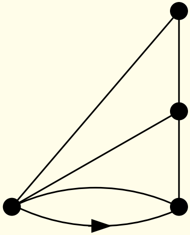
There is a well-known bijection between maps and loopless quadrangulations (i.e., maps where all faces have 4 sides)



If the map has n edges, the quadrangulation has n faces
Obs: the map is 2-connected if and only if the quadrangulation has no multiple edges

Duality and the quadrangulation

A map is self-dual if the associated quadrangulation remains the same after interchanging the colours of the vertices and reversing the root edge



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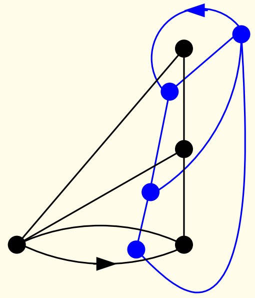

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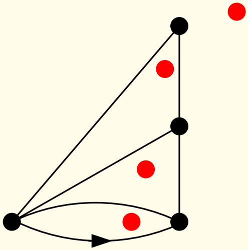

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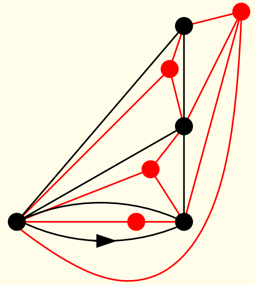

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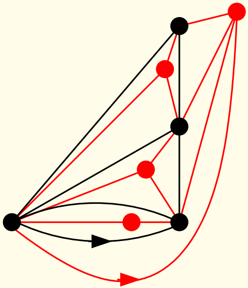

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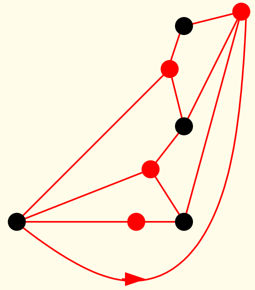

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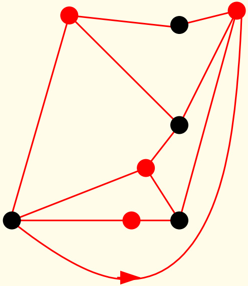

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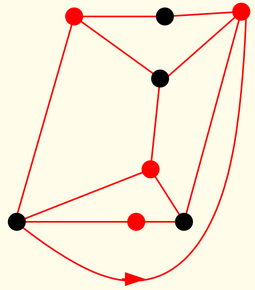

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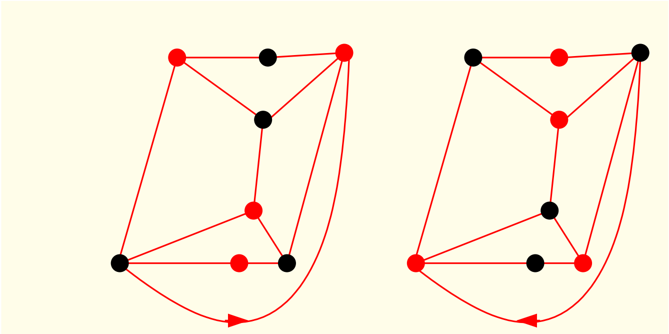

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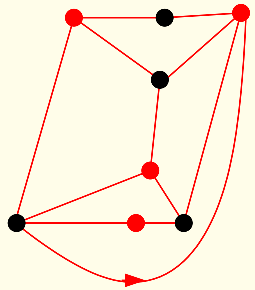

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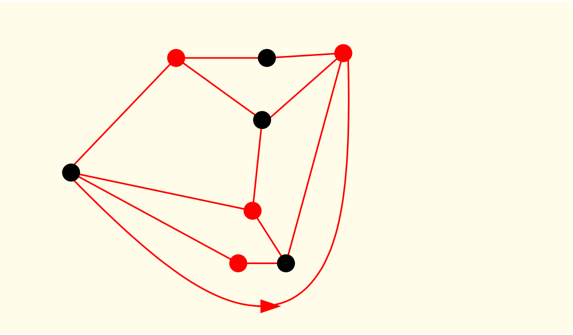

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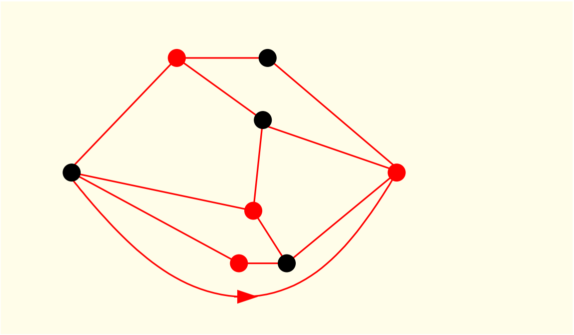

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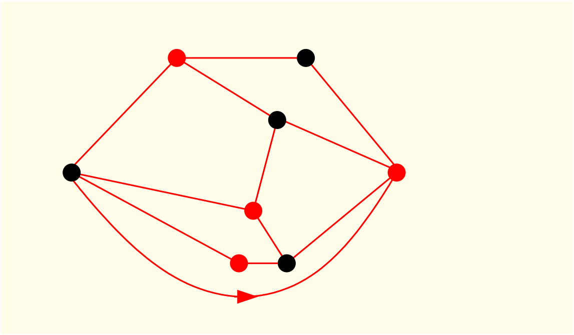

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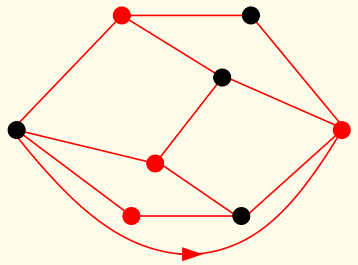

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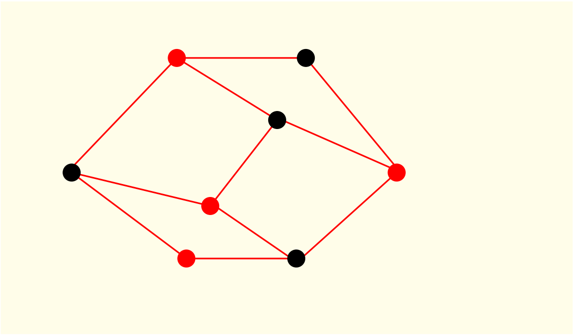

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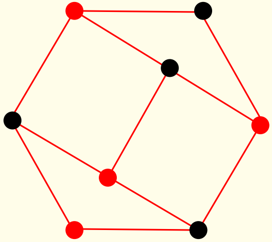

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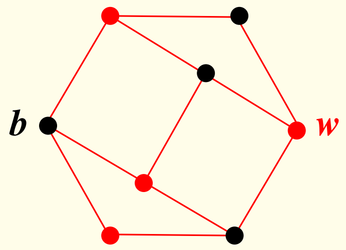

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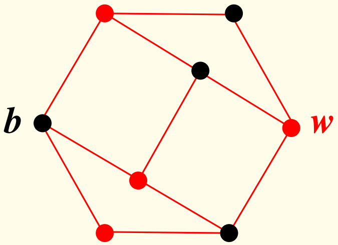

So how many quadrangulations of an hexagon are symmetric and do not have the edge *bw*?

2-connected self-dual maps

Thm (Brown 65) The number of symmetric quadrangulations of an hexagon with $2n-2$ inner faces is

$$\frac{9(3n-4)!}{(n-2)!(2n-1)!}$$

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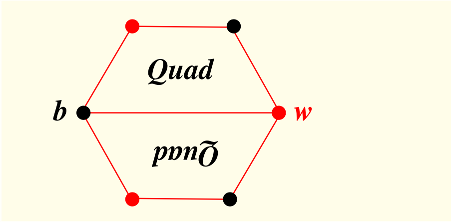

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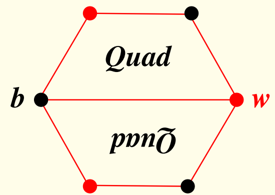

From this the formula stated follows

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Which quadrangulations come from 3-connected maps?

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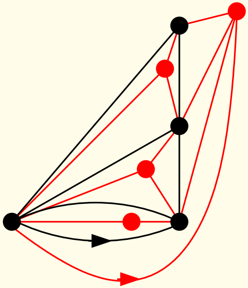

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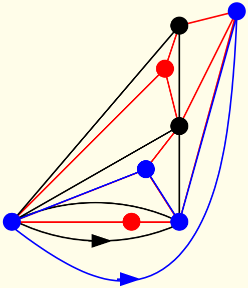

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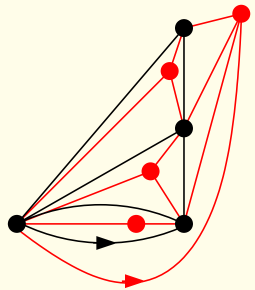

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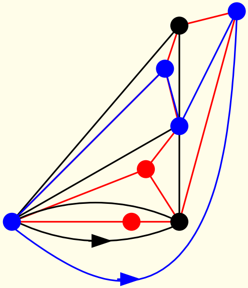

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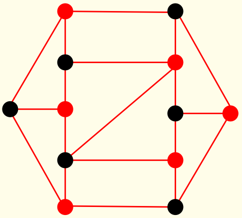

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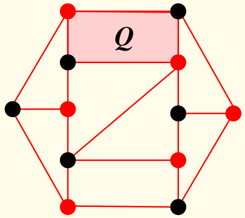

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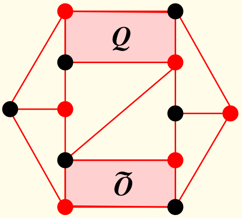

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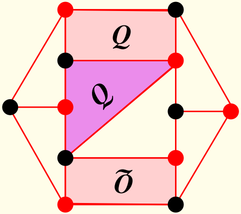

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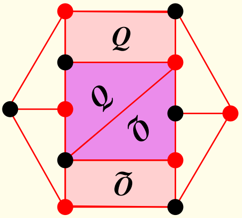

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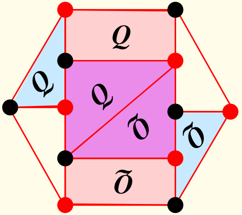

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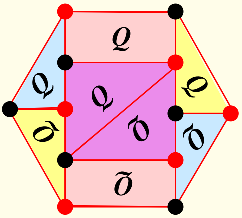

So how many symmetric quadrangulations now?

$q(n)$: the number of quadrangulations with n interior faces

$s_2(n)$: the number of quadrangulations of an hexagon that are symmetric and have $2n$ interior faces

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Then:

$$s_2(n) = \sum_{i=1}^{n} s_3(i) \sum q(n_1) \cdots q(n_i)$$

(the second $\sum$ over all solutions of $n_1 + \cdots + n_i = n,\ n_j \geq 1$)

Translating into generating functions

$Q(z)$: the GF for quadrangulations (i. e., maps) ✓

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We invert $Q(z)$ and obtain the claimed expression for $S_3(z)$

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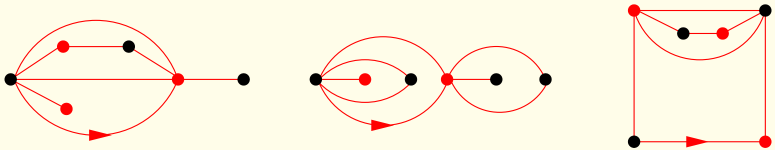

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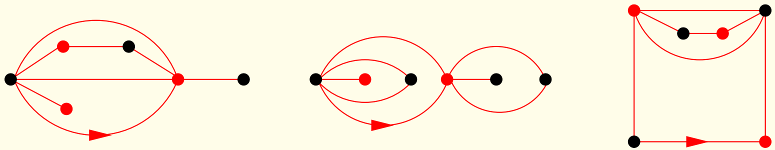

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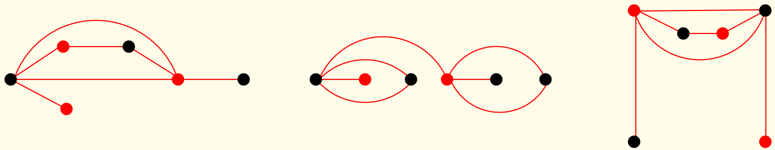

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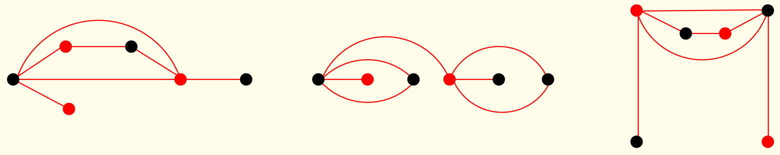

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But we can deal with it, end up with another equation involving $S_2(z)$ and arrive to the stated formula

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Given a self-dual graph, how many rootings make it a self-dual rooted map ?

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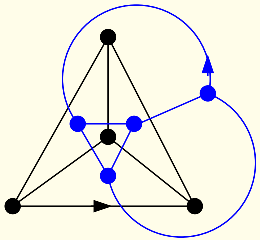

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Q1: Does the same relationship hold for 2-connected and 3-connected self-dual maps?

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Q1: Does the same relationship hold for 2-connected and 3-connected self-dual maps?

Q2: Is there a combinatorial explanation?

Muchas gracias