Enumeration of self-dual planar maps

Anna de Mier

Anna de Mier (anna.de.mier@upc.edu)
Universitat Politècnica de Catalunya

Abstract.

A planar map is an embedding of a connected planar graph in the sphere such that the surface is partitioned into simply connected regions; in other words, it is a finite cellular decomposition of the sphere into vertices, edges, and faces (0−, 1− and 2−cells, respectively). In particular, 3−connected planar maps correspond to polyhedra. Motivated by the Four Colour Problem, W. Tutte [4] launched in the 1960's the enumeration of planar maps, being today still an active topic in enumerative combinatorics.

Here we are interested in self-dual maps. The dual of a map is obtained by taking the geometric plane dual of the embedded graph; that is, placing a vertex in each face of the original map and joining two vertices if the corresponding faces shared an edge. For instance, the dual of the cube is the octahedron. We say that a map is self-dual if there is an orientation preserving homeomorphism of the sphere taking it to its dual. Also, the maps we consider are rooted at an edge; this breaks isomorphisms and makes enumeration easier. We give (very simple) formulas for the numbers of arbitrary, 2-connected and 3-connected self-dual rooted maps with $2n$ edges. Our results are obtained through the use of generating functions, combined with previous work of Brown [1]. Finally, we relate our work to that of Liskovets [3], yielding a conjecture about the number of nonisomorphic (unrooted) self-dual 3-connected maps.

References

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