The Toda system on compact surfaces

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Abstract.

This talk is devoted to the so-called Toda system on a compact surface Σ, which is assumed to have total area equal to 1:

$$\begin{cases} -\Delta u_1 = 2\rho_1 \left(\frac{h_1 e^{u_1}}{\int_\Sigma h_1 e^{u_1} dV_g} - 1 \right) - \rho_2 \left(\frac{h_2 e^{u_2}}{\int_\Sigma h_2 e^{u_2} dV_g} - 1 \right), \\ -\Delta u_2 = 2\rho_2 \left(\frac{h_2 e^{u_2}}{\int_\Sigma h_2 e^{u_2} dV_g} - 1 \right) - \rho_1 \left(\frac{h_1 e^{u_1}}{\int_\Sigma h_1 e^{u_1} dV_g} - 1 \right). \end{cases} \tag{1}$$

Here Δ is the Laplace-Beltrami operator, $\rho_1, \ \rho_2 \in \mathbb{R}$ and h_1, h_2 are smooth positive functions. This system appears naturally in geometry and mathematical physics.

Solutions of (1) correspond to critical points of certain energy functional. However, that energy functional is unbounded from below under our assumptions. A minimization argument being impossible, our proof uses min-max arguments. It is convenient to first discuss a scalar counterpart of (1):

$$-\Delta u = 2\rho \left(\frac{h\, e^u}{\int_\Sigma h\, e^u dV_g} - 1 \right). \tag{2}$$

This equation comes from the prescribed curvature problem under conformal changes of the metric.

References

[1] A. Malchiodi and D. Ruiz, A Variational Analysis of the Toda System on Compact Surfaces *Comm. Pure Applied Math.* 66 (2013), 332-371.

[2] L. Battaglia, A. Jevnikar, A. Malchiodi and D. Ruiz,. A general existence result for the Toda system on surfaces with positive genus, *preprint.*