Lineability and algebrability of the set of holomorphic functions with a given domain of existence

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Abstract.

Let E be a complex Banach space and U be an open subset of E. The space of all holomorphic functions on U will be represented by $\mathcal{H}(U)$. We prove the following results:

Theorem 1. *Let U be a domain of existence in a separable Banach space E. Then the set $\mathcal{E}(U)$ of all $f \in \mathcal{H}(U)$ whose domain of existence is U is lineable. This means that $\mathcal{E}(U)$, together with 0, contains an infinite dimensional vector subspace.*

Theorem 2. *Let U be a domain of existence in a separable Banach space E. Then the set $\mathcal{E}(U)$ of all $f \in \mathcal{H}(U)$ whose domain of existence is U is c-lineable. This means that $\mathcal{E}(U)$, together with 0, contains a vector subspace of dimension c, the cardinality of the continuum.*

(Of course Theorem 1 follows from Theorem 2, but the proof of Theorem 1 is presented in a much simpler way.)

Theorem 3. *Let U be a domain of existence in a separable Banach space E. Then the set $\mathcal{E}(U)$ of all $f \in \mathcal{H}(U)$ whose domain of existence is U is algebrable. This means that $\mathcal{E}(U)$, together with 0, contains a subalgebra which is generated by an infinite algebraically independent set.*