

# Some remarks on the solvability of non local elliptic problems with the Hardy potential.

María Medina

María Medina (maria.medina@uam.es)  
Universidad Autónoma de Madrid

**Abstract.** The aim of the talk is to study the solvability of the problem

$$(P_\mu) \begin{cases} (-\Delta)^s u - \lambda \frac{u}{|x|^{2s}} = u^p + \mu u^q & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where  $s \in (0, 1)$ ,  $\Omega \subset \mathbb{R}^N$ ,  $N > 2s$ , is an open bounded set such that  $0 \in \Omega$ ,  $\mu$  is a positive real number,  $\lambda < \Lambda_{N,s}$ , the sharp constant of the Hardy-Sobolev inequality,  $0 < q < 1$  and  $1 < p < p(\lambda, s) \equiv \frac{N+2s-2\alpha_\lambda}{N-2s-2\alpha_\lambda}$ , with  $\alpha_\lambda$  a parameter depending on  $\lambda$  and satisfying  $\alpha_\lambda \in (0, \frac{N-2s}{2})$ . We will discuss the existence and multiplicity of solutions depending on the value of  $p$ , proving in particular that  $p(\lambda, s)$  is the threshold for the existence of solution to problem  $(P_\mu)$ .

Joint work with Begoña Barrios (bego.barrios@uam.es) and Ireneo Peral (ireneo.peral@uam.es). Work partially supported by project MTM2010-18128, MICINN.

## References

- [1] M. M. FALL, *Semilinear elliptic equations for the fractional Laplacian with Hardy potential*, Preprint. arXiv:1109.5530v4 [math.AP]
- [2] R. SERVADEI AND E. VALDINOCI, *Mountain Pass solutions for non-local elliptic operators*, J. Math. Anal. Appl. **389** (2012), no. 2, 887-898.