

Inequalities of zeros of Jacobi polynomials via Sturm's theorem: Gautschi's conjectures

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Abstract. Let $x_{n,k}^{(\alpha,\beta)}$, $k = 1, \dots, n$, be the zeros of Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ arranged in decreasing order on $(-1, 1)$, where $\alpha, \beta > -1$, and $\theta_{n,k}^{(\alpha,\beta)} = \arccos x_{n,k}^{(\alpha,\beta)}$. Gautschi, in a series of recent papers, conjectured that the three sets of inequalities

$$n\theta_{n,k}^{(\alpha,\beta)} < (n+1)\theta_{n+1,k}^{(\alpha,\beta)}, \quad (n+(\alpha+\beta+1)/2)\theta_{n,k}^{(\alpha,\beta)} > (n+(\alpha+\beta+3)/2)\theta_{n+1,k}^{(\alpha,\beta)}$$

and

$$P_n^{(\alpha,\beta)}(\cos(\theta/n)) < P_{n+1}^{(\alpha,\beta)}(\cos(\theta/(n+1))),$$

hold for all $n \geq 1$, $k = 1, \dots, n$, and certain values of the parameters α and β . We establish these conjectures for large domains of the (α, β) -plane.