Trivial monodromy, bispectrality and exceptional Hermite polynomials

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Abstract.

In a classical paper [1], Duistermaat and Grünbaum characterized the class of Schrödinger operators with trivial monodromy and potentials that decay at infinity, showing they can be obtained by Darboux transformations from the free particle and that they lead to a bispectral problem: a different spectral problem where the spectral parameter plays the role of the spatial variable. Their results were extended by Oblomkov to potentials with quadratic increase at infinity, [2]. We show that bispectrality, trivial monodromy and the existence of an infinite number of polynomial eigenfunctions are intimately related, and we use this fact to prove the conjecture launched in [3] for operators of Hermite type. This conjecture states that every exceptional orthogonal polynomial system is related to a classical system by a Darboux transformation. Working out the details of this connection has allowed us to provide a complete classification of exceptional Hermite polynomials [4], studying the shape and regularity of the potential and a convenient way to index them by using partitions. A recent result of Odake [5] shows how to derive recurrence relations for the exceptional families. We use this procedure to derive recurrence relations for the exceptional Hermite families, which amount to remarkable identities between Wronskian determinants of Hermite polynomials.

References

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