Averaged alternating reflections in geodesic spaces

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Abstract. The convex feasibility problem for two sets consists in finding a point in the intersection of two nonempty closed and convex sets provided such a point exists. In Hilbert spaces there exists a wide range of algorithms that use metric projections on the sets in order to obtain sequences of points that converge weakly or in norm (under more restrictive conditions) to a solution of this problem. One of the most famous algorithms is the alternating projection method which was developed by von Neumann [1] and was recently adapted to the setting of CAT(0) spaces.

Another class of algorithms considered in this respect bases on reflections instead of projections. Given a nonempty closed and convex subset A of a Hilbert space H, the reflection of a point $x \in H$ with respect to A is the image of x by the reflection mapping $R_A = 2P_A - I$, where P_A stands for the metric projection onto A and I is the identity mapping. In this lecture we focus on the averaged alternating reflection (AAR) method which generates the following sequence for a starting point $x_0 \in H$: $x_n = T^n x_0$, where $T = \dfrac{I + R_A R_B}{2}$. We study the AAR method in geodesic spaces. Specifically, we consider spaces of constant curvature, CAT(0) spaces and the gluing of model spaces.

References

[1] von Neumann, J. Functional Operators, Vol. II. The Geometry of Orthogonal Spaces, Princeton University Press, Princeton, NJ, 1950. Ann. Math. Stud., Vol. 22. Reprint of mimeographed lecture notes first distributed in 1933.